$$
{}_{u_\ell - e}\underset{\omega}{\mathsf{E}}\,|\,{}_{\mho}\mathbb{L} = \underset{\omega}{\mathsf{E}}^{O}_{1}\,|\,{}_{\mho}\mathbb{L} \times {}_{u_\ell - e}\underset{\omega}{\mathsf{E}}^{O}_{-}\,|\,{}_{\mho}\mathbb{L} \times \sum_{\ell < i < j \leqslant r} \underset{\omega}{\mathsf{E}}^{1}_{j-i}\,|\,{}_{\mho}\mathbb{L} \times \sum_{i < j > \ell} \underset{\omega}{\mathsf{E}}^{-}_{j-i}\,|\,{}_{\mho}\mathbb{L} \times \sum_{0 \leqslant i \leqslant j > \ell} \underset{\omega}{\mathsf{E}}^{-}_{j+i}\,|\,{}_{\mho}\mathbb{L}
$$

$$
{}^{u_\ell - e}\underset{\omega}{\mathsf{E}}\,|\,{}_{\mho}\mathbb{L} = {}_{u_\ell - e}\underset{\omega}{\mathsf{E}}\,|\,{}_{\mho}\mathbb{L} \times {}^{u_\ell - e}\underset{\omega}{\mathsf{E}}^{O}_{-}\,|\,{}_{\mho}\mathbb{L} \times \sum_{1 \leqslant i < j \leqslant \ell} \underset{\omega}{\mathsf{E}}^{\varkappa}_{j-i}\,|\,{}_{\mho}\mathbb{L} \times \sum_{0 \leqslant i \leqslant j \leqslant \ell} \underset{\omega}{\mathsf{E}}^{\varkappa}_{j+i}\,|\,{}_{\mho}\mathbb{L}
$$

$$
{}_{u_\ell - e}\underset{\omega}{\mathsf{E}}^{O}_{-}\,|\,{}_{\mho}\mathbb{L} = \mathbb{K}\frac{e_k - \overset{*}{e}_k}{k > \ell}
$$

$$
{}^{u_\ell - e}\underset{\omega}{\mathsf{E}}^{O}_{-}\,|\,{}_{\mho}\mathbb{L} = \mathbb{K}\frac{e_k - \overset{*}{e}_k}{k \leqslant \ell}
$$

$$
\left(u_\ell - e\right)\delta = -\sum_{k > \ell} e_k \delta = 0
$$

$$
\left(u_\ell - e\right) X^{-}_{e_k} = e_k - \left(u_\ell - e\right) \overset{*}{e}_k \left(u_\ell - e\right) = \begin{cases} e_k & k \leqslant \ell \\ 0 & k > \ell \end{cases}
$$

$$
\left(u_\ell - e\right) \overbrace{X^{-}_{a} + \varepsilon i \sharp j \underbrace{\overset{*}{a} e_j - \overset{*}{e}_j a}} = a - \left(u_\ell - e\right) \overset{*}{a} \left(u_\ell - e\right) + \varepsilon i \sharp j \Big(\left(u_\ell - e\right) \overset{*}{a} e_j - \left(u_\ell - e\right) \overset{*}{e}_j a \Big) =
$$

$$
\begin{cases}
a - \overset{*}{a} + 2\varkappa\left(-\overset{*}{a}/2 + a/2\right) = \left(1 + \varkappa\right)\left(1 - \varepsilon\right) a & \ell < i < j \\
a - \overset{*}{a} + \varkappa\left(-\overset{*}{a} + a\right) = \left(1 + \varkappa\right)\left(1 - \varepsilon\right) a & \ell < i = j \\
a + 2\varkappa a/2 = \left(1 + \varkappa\right) a & 0 \leqslant i \leqslant \ell < j \\
a & 0 \leqslant i \leqslant j \leqslant \ell
\end{cases}
$$

$$\operatorname{tr}\ \ltimes_{\mathcal{P}} \sum_{k} \lambda^{k}\, \underbrace{e_k - \overset{*}{e}_k} = \operatorname{tr}\ \ltimes_{\mathcal{Q}} \sum_{k} \lambda^{k}\, \underbrace{e_k - \overset{*}{e}_k} = -\underbrace{a\,(r+1-\ell) + 2\,(1+b)}\,\underbrace{\lambda^{\ell+1} + \cdots + \lambda^{r}}$$

$$-\ \mathrm{LHS}\ = \sum_{\ell < i < j} \underbrace{\lambda^{i} - \lambda^{j}} a + \sum_{i < j > \ell} \underbrace{\lambda^{j} - \lambda^{i}} a + \sum_{i < j > \ell} \underbrace{\lambda^{i} + \lambda^{j}} a + \sum_{\ell < j} 2\,\lambda^{j} + \sum_{\ell < j} \lambda^{j}\, b$$

$$= \sum_{i \leqslant \ell < j} \underbrace{\lambda^{j} - \lambda^{i}} a + \sum_{i \leqslant \ell < j} \underbrace{\lambda^{i} + \lambda^{j}} a + \sum_{\ell < i < j} \underbrace{\lambda^{i} + \lambda^{j}} a + 2 \sum_{\ell < j} \lambda^{j}\ (1+b)$$

$$= \sum_{i \leqslant \ell < j} 2\,\lambda^{j} a + \sum_{\ell < j} (r - \ell - 1)\ \lambda^{j} a + 2 \sum_{\ell < j} \lambda^{j}\ (1+b) = \sum_{\ell < j} \lambda^{j}\ \underbrace{2a\ell + a\,(r - \ell - 1) + 2\,(1+b)}$$

$$= \sum_{\ell < j} \lambda^{j}\ \underbrace{a\,(r + \ell - 1) + 2\,(1+b)}$$

$$\operatorname{tr}\ \ltimes_{\mathcal{Q}} \sum_{k} \lambda^{k}\, \underbrace{e_k - \overset{*}{e}_k} - \operatorname{tr}\ \ltimes_{\mathcal{P}} \sum_{k} \lambda^{k}\, \underbrace{e_k - \overset{*}{e}_k} = \sum_{i < j \leqslant \ell} \varkappa \underbrace{\lambda^{j} - \lambda^{i}} a + \sum_{0 \leqslant i \leqslant j \leqslant \ell} \varkappa \underbrace{\lambda^{i} + \lambda^{j}} a = 0$$