$$D \underset{\omega}{\overset{2}{\triangle}} C^{\chi} \xleftarrow[\text{metrep}]{C^{\chi}} G \ltimes D \underset{\omega}{\triangle} C^{\chi}$$

$$^{z}\overbrace{C^{\chi}_{g}\text{٩}} = {}^{z}C^{\chi}_{g}\,{}^{zg}\text{٩}$$

$$\begin{array}{ccc} C^{\chi} & \xleftarrow{\;{}^{zg}()\;} & D \underset{\omega}{\overset{2}{\triangle}} C^{\chi} \\ \downarrow {}^{z}C^{\chi}_{g} & & \downarrow C^{\chi}_{g} \\ C^{\chi} & \xleftarrow{\;{}^{z}()\;} & D \underset{\omega}{\overset{2}{\triangle}} C^{\chi} \end{array}$$

$$C_{g}\,\overbrace{C_{\acute{g}}\text{٩}} = C_{g\acute{g}}\,\text{٩}$$

$$^{z}\text{LHS} = {}^{z}C_{g}\;{}^{zg}\overbrace{C_{\acute{g}}\text{٩}} = {}^{z}C_{g}\;{}^{zg}C_{\acute{g}}\;{}^{\underbrace{zg\acute{g}}}\text{٩} = {}^{z}C_{g\acute{g}}\;{}^{\underbrace{zg\acute{g}}}\text{٩} = {}^{z}\text{RHS}$$

$$\overbrace{C_{g}\text{٩}} \Join \overbrace{C_{g}\text{ꝗ}} = \text{٩} \underset{\chi}{\Join} \text{ꝗ}$$

$$\text{LHS} = \int_{dz}^{D} {}^{z}\overbrace{C_{g}\text{٩}} \Join \underbrace{{}^{z}C_{z}\,{}^{z}\overbrace{C_{g}\text{ꝗ}}} = \int_{dz}^{D} \underbrace{{}^{z}C_{g}\,{}^{zg}\text{٩}} \Join \overbrace{{}^{z}C_{z}\underbrace{{}^{z}C_{g}\,{}^{zg}\text{ꝗ}}}$$

$$= \int_{dz}^{D} {}^{zg}\text{٩} \Join \overbrace{{}^{z}\overset{*}{C_{g}}\,{}^{z}C_{z}\,{}^{z}C_{g}\,{}^{zg}\text{ꝗ}} = \int_{dz}^{D} \overbrace{{}^{zg}\text{٩}} \Join \overbrace{{}^{zg}C_{zg}\,{}^{zg}\text{ꝗ}} = \int_{dw}^{D} {}^{w}\text{٩} \Join \overbrace{{}^{w}C_{w}\,{}^{w}\text{ꝗ}} = \text{RHS}$$

GCD

$$ {}^{D}\underset{\varpi}{\overset{2}{◺}} C^{\chi} ⧓ T^{\nu} \xleftarrow[\text{metrep}]{g^{\chi\nu}_{CT}} G \ltimes {}^{D}\underset{\varpi}{◺} C^{\chi} ⧓ T^{\nu} : {}^{z}\overbrace{g^{\chi\nu}_{CT}\,\text{ᑫ}} = {}^{z}_{C}g^{\chi}\; {}^{zg}\text{ᑫ}\; {}^{z}_{T}g^{\nu} $$

$$ g \underset{\nu}{\overset{\chi}{⋉}} \overbrace{\acute{g} \underset{\nu}{\overset{\chi}{⋉}} \text{ᑫ}} = \underbrace{g\acute{g}} \underset{\nu}{\overset{\chi}{⋉}} \text{ᑫ} $$

$$ {}^{z}\text{LHS} = {}^{z}_{C}g\; {}^{zg}\overbrace{\acute{g} \ltimes \text{ᑫ}} = {}^{z}_{C}g\; {}^{zg}_{C}\acute{g}\; {}^{\underbrace{zg\acute{g}}}\text{ᑫ} = {}^{z}\underbrace{\overbrace{g\acute{g}}}\; {}^{\underbrace{zg\acute{g}}}\text{ᑫ} = {}^{z}\text{RHS} $$

$$ \overbrace{g \ltimes \text{ᑫ}} ⧗ \overbrace{g \ltimes \text{ꝗ}} = \text{ᑫ} \underset{\nu}{⧗} \text{ꝗ} $$

$$ \overbrace{g \underset{\nu}{⋉} \text{ᑫ}} \underset{\nu}{⧗} \overbrace{g \underset{\nu}{⋉} \text{ꝗ}} = \int_{dz}^{D} {}^{z}\Delta^{\nu}_{z}\, \underbrace{{}^{o}_{C}\mathfrak{g}_{z}\; {}^{z}\overbrace{g \underset{\nu}{⋉} \text{ᑫ}}} ⧗ \underbrace{I\, {}^{o}_{C}\mathfrak{g}_{z}\; {}^{z}\overbrace{g \underset{\nu}{⋉} \text{ꝗ}}} = \int_{dz}^{D} {}^{z}_{T}\overset{*}{g}{}^{\nu}\; {}^{z}\Delta^{\nu}_{z}\; {}^{z}_{T}g^{\nu}\; \underbrace{{}^{o}_{C}\mathfrak{g}_{z}\, {}^{z}_{C}g\; {}^{zg}\text{ᑫ}} ⧗ \underbrace{I\, {}^{o}_{C}\mathfrak{g}_{z}\, {}^{z}_{C}g\; {}^{zg}\text{ꝗ}} $$

$$ = \int_{dz}^{D} {}^{zg}\Delta^{\nu}_{zg}\, \underbrace{{}^{o}_{C}\mathfrak{g}_{zg}\; {}^{zg}\text{ᑫ}} ⧗ \underbrace{I\, {}^{o}_{C}\mathfrak{g}_{zg}\; {}^{zg}\text{ꝗ}} = \int_{dz}^{D} {}^{z}\Delta^{\nu}_{z}\, \underbrace{{}^{o}_{C}\mathfrak{g}_{z}\; {}^{z}\text{ᑫ}} ⧗ \underbrace{I\, {}^{o}_{C}\mathfrak{g}_{z}\; {}^{z}\text{ꝗ}} = \text{RHS} $$

GCD

$$
{}^{-1}_{\;C}g^{\chi}\;\; {}_{C}G^{-1}_{w}\,\blacktriangleleft = {}_{C}G^{-1}_{wg}\;\; {}^{w*}_{C}g\,\blacktriangleleft
$$

$$
\begin{array}{ccc}
D\,{\overset{2}{\underset{\omega}{\triangle}}}\,C^{\chi}\bowtie T^{\nu} & \xleftarrow[\Delta^{-\nu}_{w}]{{}_{C}G^{-\chi}_{w}} & C^{\chi}\bowtie T^{\nu} \\
\Big\uparrow {\scriptstyle g^{\chi}_{C}\,\mid\, g^{\nu}_{T}} & & \Big\downarrow {\scriptstyle {}^{w*}_{C}g^{\chi}\,\mid\, {}^{w*}_{T}g^{\nu}} \\
D\,{\overset{2}{\underset{\omega}{\triangle}}}\,C^{\chi}\bowtie T^{\nu} & \xleftarrow[\Delta^{-\nu}_{wg}]{{}_{C}G^{-\chi}_{wg}} & C^{\chi}\bowtie T^{\nu}
\end{array}
$$

$$
{}^{zg}\text{LHS} = {}^{zg}_{C}g^{-1}\;\; {}^{z}\overbrace{{}_{C}G^{-1}_{w}\,\blacktriangleleft} = {}^{z}_{C}g^{-1}\;\; {}^{z}_{C}G^{-1}_{w}\,\blacktriangleleft = {}^{zg}_{C}G^{-1}_{wg}\;\; {}^{w*}_{C}g\,\blacktriangleleft = {}^{zg}\text{RHS}
$$

$$
{}_{C}G^{-\chi} = \int\limits_{dx}^{D} {}^{o}_{C}G^{-\chi}_{x}\;\; {}^{x}_{C}G_{x}\;\; {}^{x}_{C}G^{-\chi}_{o} = \int\limits_{dx}^{D} {}^{o}_{C}G^{-\chi}_{x}\;\; {}^{o*}_{C}\mathfrak{g}_{z}\,\mathcal{J}\;\; {}^{o}_{C}\mathfrak{g}_{z}\;\; {}^{x}_{C}G^{-\chi}_{o}
$$

$$
\mathcal{J} \underset{\text{bij}}{\longmapsto} \bar{\mathcal{J}} = \int\limits_{dx}^{D} {}^{o}_{C}G^{-1}_{x}\;\; {}^{x}_{C}G_{x}\;\; {}^{x}_{C}G^{-1}_{o}
$$

$$
{}^{z}_{C}G^{-\chi}_{w} = \int\limits_{dx}^{D} {}^{z}_{C}G^{-\chi}_{x}\;\; {}^{x}_{C}G_{x}\;\; {}^{x}_{C}G^{-\chi}_{w}
$$

$$
\int\limits_{dx}^{D} {}^{z}_{C}G^{-\chi}_{x}\;\; {}^{x}_{C}G_{x}\;\; {}^{x}_{C}G^{-\chi}_{z} = \int\limits_{dx}^{D} {}^{o\mathfrak{g}_{z}}_{C}G^{-\chi}_{x\mathfrak{g}_{z}}\;\; {}^{x\mathfrak{g}_{z}}_{C}G_{x\mathfrak{g}_{z}}\;\; {}^{x\mathfrak{g}_{z}}_{C}G^{-\chi}_{o\mathfrak{g}_{z}} = \int\limits_{dx}^{D} \overbrace{{}^{x*}_{C}\mathfrak{g}_{z}\,{}^{o}_{C}G^{\chi}_{x}\,{}^{o}_{C}\mathfrak{g}_{z}}^{-1}\;\; {}^{x*}_{C}\mathfrak{g}_{z}\;\; {}^{x}_{C}G_{x}\;\; {}^{x}_{C}\mathfrak{g}_{z}\;\; \overbrace{{}^{o*}_{C}\mathfrak{g}_{z}\,{}^{x}_{C}G^{\chi}_{o}\,{}^{x}_{C}\mathfrak{g}_{z}}^{-1}
$$

$$
= {}^{o}_{C}\mathfrak{g}^{-1}_{z} \int\limits_{dx}^{D} {}^{o}_{C}G^{-\chi}_{x}\;\; {}^{x}_{C}G_{x}\;\; {}^{x}_{C}G^{-\chi}_{o}\;\; {}^{o*}_{C}\mathfrak{g}^{-1}_{z} = {}^{o}_{C}\mathfrak{g}^{-1}_{z}\;\; {}_{C}G^{-\chi}\;\; {}^{o*}_{C}\mathfrak{g}^{-1}_{z} = \overbrace{{}^{o*}_{C}\mathfrak{g}_{z}\,{}_{C}G^{\chi}\,{}^{o}_{C}\mathfrak{g}_{z}}^{-1} = {}^{z}_{C}G^{-\chi}_{z}
$$

$${}^{z}\mathsf{q} \in C^{\chi} \xleftarrow[\text{stetlin}]{{}^{z}\mathcal{E}} {}^{D}\underset{w}{\overset{2}{\triangle}} C^{\chi} \ni \mathsf{q}$$

$$\text{kernel } C^{\chi} \xleftarrow[\text{lin}]{{}^{z}_{C}G^{\chi}_{w}} C^{\chi}$$

$$\mathsf{p} \bowtie {}^{z}\mathsf{q} = \underbrace{{}_{C}G^{-\chi}_{z}\mathsf{p}} \bowtie \mathsf{q}$$

$${}^{z}\mathsf{q} = \int_{dw}^{D} {}^{z}_{C}G^{-\chi}_{w}\, {}^{w}_{C}G_{w}\, {}^{w}\mathsf{q} \text{ repr kernel}$$

$$\mathsf{q} = {}_{C}G^{-\chi}_{w}\, \not\mathsf{p}$$

$$\underbrace{{}_{C}G^{-\chi}_{z}\mathsf{p}} \bowtie \mathsf{q} = \underbrace{{}_{C}G^{-\chi}_{z}\mathsf{p}} \bowtie \underbrace{{}_{C}G^{-\chi}_{w}\not\mathsf{p}} = \int_{dx}^{D} {}^{x}\overbrace{{}_{C}G^{-\chi}_{z}\mathsf{p}} \bowtie \underbrace{{}^{x}_{C}G_{x}\, {}^{x}\overbrace{{}_{C}G^{-\chi}_{w}\not\mathsf{p}}} = \int_{dx}^{D} \underbrace{{}^{x}_{C}G^{-\chi}_{z}\mathsf{p}} \bowtie \underbrace{{}^{x}_{C}G_{x}\, {}^{x}_{C}G^{-\chi}_{w}\not\mathsf{p}}$$

$$= \int_{dx}^{D} \mathsf{p} \bowtie \underbrace{{}^{z}_{C}G^{-\chi}_{x}\, {}^{x}_{C}G_{x}\, {}^{x}_{C}G^{-\chi}_{w}\not\mathsf{p}} = \mathsf{p} \bowtie \overbrace{\underbrace{\int_{dx}^{D} {}^{z}_{C}G^{-\chi}_{x}\, {}^{x}_{C}G_{x}\, {}^{x}_{C}G^{-\chi}_{w}}\not\mathsf{p}} = \mathsf{p} \bowtie \overbrace{{}^{z}_{C}G^{-\chi}_{w}\mathsf{p}} = \mathsf{p} \bowtie {}^{z}\mathsf{q}$$

$${}^{w*}_{C}\breve{g}\, {}^{z}_{C}G^{\chi}_{w}\, {}^{z}_{C}g = {}^{zg}_{C}G^{\chi}_{wg}$$

$$\int_{dz}^{D} \underbrace{{}^{z}_{C}g\, {}^{zg}_{C}G^{-\chi}_{wg}\, {}^{w*}_{C}\breve{g}\mathsf{p}} \bowtie \underbrace{{}^{z}_{C}G_{z}\, {}^{z}\overbrace{g \ltimes \mathsf{q}}} = \int_{dz}^{D} \underbrace{{}^{zg}_{C}G^{-\chi}_{wg}\, {}^{w*}_{C}\breve{g}\mathsf{p}} \bowtie \underbrace{{}^{z*}_{C}\breve{g}\, {}^{z}_{C}G_{z}\, {}^{z}\overbrace{g \ltimes \mathsf{q}}} = \int_{dz}^{D} \underbrace{{}^{zg}_{C}G^{-\chi}_{wg}\, {}^{w*}_{C}\breve{g}\mathsf{p}} \bowtie \underbrace{{}^{zg}_{C}G_{zg}\, {}^{z}_{C}g^{-1}\, {}^{z}\overbrace{g \ltimes \mathsf{q}}}$$

$$= \int_{dz}^{D} \underbrace{{}^{zg}_{C}G^{-\chi}_{wg}\, {}^{w*}_{C}\breve{g}\mathsf{p}} \bowtie \underbrace{{}^{zg}_{C}G_{zg}\, {}^{zg}\mathsf{q}} \underset{\text{inv}}{=} \int_{dz}^{D} \underbrace{{}^{z}_{C}G^{-\chi}_{wg}\, {}^{w*}_{C}\breve{g}\mathsf{p}} \bowtie \underbrace{{}^{z}_{C}G_{z}\, {}^{z}\mathsf{q}} = \underbrace{{}_{C}G^{-\chi}_{wg}\, {}^{w*}_{C}\breve{g}\mathsf{p}} \bowtie \mathsf{q}$$

$$= \overbrace{{}^{w*}_{C}\breve{g}\mathsf{p}} \bowtie {}^{wg}\mathsf{q} = \mathsf{p} \bowtie \underbrace{{}^{w}_{C}g\, {}^{wg}\mathsf{q}} = \mathsf{p} \bowtie {}^{w}\overbrace{g \ltimes \mathsf{q}} = \underbrace{{}_{C}G^{-\chi}_{w}\mathsf{p}} \bowtie \underbrace{g \ltimes \mathsf{q}} = \int_{dz}^{D} \underbrace{{}^{z}_{C}G^{-\chi}_{w}\mathsf{p}} \bowtie \underbrace{{}^{z}_{C}G_{z}\, {}^{z}\overbrace{g \ltimes \mathsf{q}}}$$

$$\int\limits_{dx}^{D} {}^{z}_{C}G^{-\chi}_{x}\,{}^{x}_{C}G_{x}\,{}^{x}_{C}G^{-\chi}_{z} = {}^{o}_{C}g^{-1}_{\;z}\underbrace{\left(\int\limits_{dx}^{D} {}^{o}_{C}G^{-\chi}_{x}\,{}^{x}_{C}G_{x}\,{}^{x}_{C}G^{-\chi}_{o}\right)}_{=C}{}^{o}_{C}g^{-*}_{\;z}$$

$$\int\limits_{dx}^{D} {}^{z}_{C}G^{-\chi}_{x}\,{}^{x}_{C}G_{x}\,{}^{x}_{C}G^{-\chi}_{z} = \int\limits_{dx}^{D} {}^{og}_{C}G^{-\chi}_{x}\,{}^{x}_{C}G_{x}\,{}^{x}_{C}G^{-\chi}_{og} = \int\limits_{dx}^{D} {}^{og}_{C}G^{-\chi}_{xg}\,{}^{xg}_{C}G_{xg}\,{}^{xg}_{C}G^{-\chi}_{og} = \int\limits_{dx}^{D} {}^{o}_{C}g^{-1}\,{}^{o}_{C}G^{-\chi}_{x}\,{}^{x}_{C}g^{-*}\,\overbrace{{}^{x}_{C}g^{-1}\,{}^{x}_{C}G^{-\chi}_{x}\,{}^{x}_{C}g^{-*}}\,{}^{x}_{C}g^{-1}\,{}^{x}_{C}G^{-\chi}_{o}\,{}^{o}_{C}g^{-*}$$

$$= \int\limits_{dx}^{D} {}^{o}_{C}g^{-1}\,{}^{o}_{C}G^{-\chi}_{x}\,{}^{x}_{C}g^{-*}\,{}^{x}_{C}g\,{}^{x}_{C}G_{x}\,{}^{x}_{C}g^{*}\,{}^{x}_{C}g^{-1}\,{}^{x}_{C}G^{-\chi}_{o}\,{}^{o}_{C}g^{-*} = \int\limits_{dx}^{D} {}^{o}_{C}g^{-1}\,{}^{o}_{C}G^{-\chi}_{x}\,{}^{x}_{C}G_{x}\,{}^{x}_{C}G^{-\chi}_{o}\,{}^{o}_{C}g^{-*} = {}^{o}_{C}g^{-1}\underbrace{\int\limits_{dx}^{D} {}^{o}_{C}G^{-\chi}_{x}\,{}^{x}_{C}G_{x}\,{}^{x}_{C}G^{-\chi}_{o}}\,{}^{o}_{C}g^{-*}$$