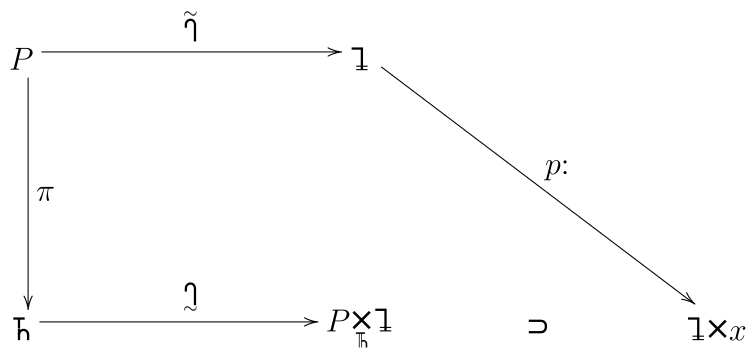
$$ {}^{p\pi}\underset{\sim}{\text{ʅ}} = p^{p}\tilde{\text{ʔ}} $$

$$ P \leftarrow \mathbb{h} \times P $$

$$ \text{ℸ} \leftarrow \mathbb{h} \ltimes \text{ℸ} $$

$$ P \underset{\mathbb{h}}{\times} \underline{\text{ℸ}} = \begin{cases} p{:}\text{Ч} \sim \mathbb{h} p{:}\mathbb{h} \ltimes \text{Ч} \\ p \in P{:}\text{Ч} \in \underline{\text{ℸ}} \end{cases} $$

$$ p{:}\text{Ч} \in \text{ℸ} \times x \underset{\exists}{\overset{p:}{\longleftarrow}} \text{ℸ} \ni \text{Ч} $$



$$ \begin{cases} p^{p}\tilde{\text{ʔ}} = {}^{x}\underset{\sim}{\text{ʅ}} \\ {}^{p}\tilde{\text{ʔ}} = p^{-1\,x}\underset{\sim}{\text{ʅ}} \end{cases} $$

$$ \underline{\text{ℸ}} \ni \begin{cases} {}^{\gamma_x}\tilde{\text{ʔ}} = {}_{1/2}^{\;\;x}B_x{}^{x}\text{ʔ} \\ {}^{g}\tilde{\text{ʔ}} = {}^{o}\underline{g}^{og}\text{ʔ} \end{cases} \quad \Longrightarrow \; {}^{kg}\tilde{\text{ʔ}} = {}^{o}\underline{kg}\;{}^{okg}\text{ʔ} = {}^{o}\underline{k}\;{}^{ok}\underline{g}\;{}^{og}\text{ʔ} = {}^{o}\underline{k}\;{}^{o}\underline{g}\;{}^{og}\text{ʔ} = {}^{o}\underline{k}\;{}^{g}\tilde{\text{ʔ}} $$

$$ \underline{\text{ℸ}} \times (og) \ni \begin{cases} {}^{og}\underline{\text{ʅ}} = {}^{g}\tilde{\text{ʔ}}{:}g = {}^{o}\underline{g}^{og}\text{ʔ}{:}g \\ {}^{x}\underline{\text{ʅ}} = {}^{\gamma_x}\tilde{\text{ʔ}}{:}\gamma_x = {}_{1/2}^{\;\;x}B_x{}^{x}\text{ʔ}{:}\gamma_x \end{cases} $$

$$ {}^{x}\overbrace{\nabla_{\dot x}\eta} = \underbrace{\overbrace{\dot p - \underbrace{\dot p^{p}\P\ltimes}_{p}}{}^{p}\underline{\tilde{\eta}}\, p} $$

$$ {}^{p}\overline{\nabla_{\dot p^{p}\underline{\pi}}\eta} = \dot p\, {}^{p}\underline{\tilde{\eta}} - {}^{p}\overbrace{\underbrace{\dot p^{p}\P}\ltimes\tilde{\eta}} $$

$$ \text{LHS} = \overbrace{\dot p - \underbrace{\dot p^{p}\P\ltimes}_{p}}^{\text{horiz lift of } \dot p^{p}\underline{\pi} = \dot x}\ {}^{p}\underline{\tilde{\eta}} = \dot p\, {}^{p}\underline{\tilde{\eta}} - \underbrace{\dot p^{p}\P\ltimes}_{p}{}^{p}\underline{\tilde{\eta}} = \text{RHS} $$