

$$\begin{array}{ccc}
\mathbb{C}\mathbb{Z} & \xrightarrow{\pi} & \mathbb{R}\mathbb{Z} \\
\downarrow \gamma & & \downarrow \gamma \\
\mathbb{C}\mathbb{Z} & \xrightarrow{\pi} & \mathbb{R}\mathbb{Z}
\end{array}$$

$$\left\{ \begin{array}{l} x \in \mathbb{R}\mathbb{Z} \\ y \in \mathbb{R}\mathbb{Z}i \end{array} \right. \Rightarrow y\Lambda \in \pi^{-1}(0) \subset \mathbb{C}\mathbb{Z} \Rightarrow w = y\Lambda\gamma_x \in \mathbb{C}\mathbb{Z} \Rightarrow w = u + v \left\{ \begin{array}{l} u = \overset{\#}{u} \in \mathbb{R}\mathbb{Z} \\ v = -\overset{\#}{v} \in \mathbb{R}\mathbb{Z}i \end{array} \right.$$

$$\left\{ \begin{array}{l} u = \frac{w + \overset{\#}{w}}{2} \\ v = \frac{w - \overset{\#}{w}}{2} \end{array} \right. \in \mathbb{C}\mathbb{Z} \text{ conv}$$

$$x:y \in \mathbb{R}\mathbb{Z} \times \pi^{-1}(0) \xrightarrow{\mathcal{J}} \mathbb{C}\mathbb{Z} \ni y\gamma_x = \overset{y}{y}\gamma_x = y_{-x} {}^x B_x^{1/2} + x$$

$$y\overset{\#}{\gamma}_x = \overline{\overset{\#}{y}\gamma_x} = \overline{y_{-x} {}^x B_x^{1/2} + x} = \overset{\#}{y}_{-x} {}^x B_x^{1/2} + x = \overset{\#}{x:y} \mathcal{J}$$

$${}_{0:y} \mathcal{J} = y\gamma_0 = y$$

$$\overset{\#}{x:y} \mathcal{J} = \overset{\#}{x} + \overset{\#}{y} - y\overset{\#}{x}y$$

$$\pi^{-1}(x) = \left\{ \begin{array}{l} y\gamma_x \\ x \in \mathbb{R}\mathbb{Z} \\ y \in \pi^{-1}(0) \end{array} \right.$$

$$\pi(y\gamma_x) = \pi(\overset{y}{y}\gamma_x) = \pi(y)\gamma_x = {}_0\gamma_x = x$$

$$\int_{dw} \int_{\mathbb{C}\Delta_w^{-p}} w \int_{\text{Leb}} \int_{\mathbb{R}\mathbb{Z}} \int_{\pi^{-1}(0)} \overline{\gamma_{-}(x:y)} \overset{y\gamma_x}{\mathbb{C}\Delta_{y\gamma_x}^{-p}} \overset{y\gamma_x}{y\gamma_x} \mathcal{J}$$

$${}^z\Delta_z^\nu \mathbb{C} \xrightarrow{{}^z\overline{\Delta}^{-\nu}} {}^z\Delta_z^\nu G_{\mathbb{R}} \text{ inv}$$

$${}^{zg}\Delta_{0g}^\nu = {}^z\underline{g}^{\nu/p} \mathbb{C} \xrightarrow{{}^z\Delta_0^\nu} {}^{0g}\Delta_{0g}^\nu = {}^z\underline{g}^{\nu/p} \mathbb{C} \xrightarrow{{}^z\overline{g}^{\nu/p}} {}^{0g}\Delta_{0g}^\nu = {}^z\underline{g}^{\nu/p} \mathbb{C} \xrightarrow{{}^z\overline{g}^{\nu/p}} {}^{0g}\Delta_{0g}^\nu = {}^z\underline{g}^{2\nu/p} \mathbb{C} \Rightarrow \begin{cases} {}^{0g}\Delta_{0g}^\nu = {}^{0g}\Delta_{0g}^{\nu/2} \\ {}^z\underline{g}^{\nu/p} \mathbb{C} = {}^{zg}\Delta_{0g}^\nu \mathbb{C} \Delta_{0g}^{-\nu/2} \end{cases}$$

$${}^{zg}\Delta_{zg}^\nu = {}^z\underline{g}^{\nu/p} \mathbb{C} \xrightarrow{{}^z\Delta_z^\nu} {}^z\underline{g}^{\nu/p} \mathbb{C} \xrightarrow{{}^z\overline{g}^{\nu/p}} {}^{zg}\Delta_{0g}^\nu \mathbb{C} \Delta_{0g}^{-\nu} \mathbb{C} \xrightarrow{{}^z\Delta_{zg}^\nu} {}^{zg}\Delta_{zg}^\nu \mathbb{C} \xrightarrow{{}^z\Delta_z^\nu}$$

$${}^{zg}\Delta_{zg}^\nu \mathbb{C} \Delta_{\overline{z}g}^{-\nu/2} = {}^{zg}\Delta_{0g}^\nu \mathbb{C} \Delta_{0g}^{-\nu} \mathbb{C} \Delta_{zg}^\nu \mathbb{C} \Delta_z^\nu \mathbb{C} \Delta_{0g}^{-\nu/2} \mathbb{C} \Delta_{0g}^{\nu/2} \mathbb{C} \Delta_{\overline{z}g}^{-\nu/2} \mathbb{C} \Delta_{\overline{z}}^{-\nu/2} = {}^{0g}\Delta_{0g}^{-\nu/2} \mathbb{C} \Delta_{zg}^\nu \mathbb{C} \Delta_z^\nu \mathbb{C} \Delta_{\overline{z}}^{-\nu/2}$$

$$\zeta_{\mathfrak{g}_w} \Delta_{\zeta_{\mathfrak{g}_w}}^\nu \mathbb{C} \Delta_{\overline{\zeta}_{\mathfrak{g}_w}}^{-\nu/2} = \zeta_{\mathfrak{g}_w} \Delta_{\zeta_{\mathfrak{g}_w}}^\nu \mathbb{C} \xrightarrow{\zeta_{\mathfrak{g}_w} \overline{\Delta}^{-\nu}} \overline{\zeta}_{\mathfrak{g}_w} \Delta_{\zeta_{\mathfrak{g}_w}}^{\nu/2} \mathbb{C} = \zeta_{\mathfrak{g}_w} \Delta_{\zeta_{\mathfrak{g}_w}}^\nu \mathbb{C} \xrightarrow{\zeta_{\mathfrak{g}_w} \overline{\Delta}^{-\nu}} \overline{\zeta}_{\mathfrak{g}_w} \Delta_{\zeta_{\mathfrak{g}_w}}^{\nu/2} \mathbb{C} = \zeta_{\mathfrak{g}_w} \Delta_{\zeta_{\mathfrak{g}_w}}^\nu \mathbb{C} \Delta_{\overline{\zeta}_{\mathfrak{g}_w}}^{-\nu/2} \mathbb{C} \Delta_{\zeta_{\mathfrak{g}_w}}^{-\nu/2} \mathbb{C} \Delta_{\overline{\zeta}_{\mathfrak{g}_w}}^{-\nu/2} \mathbb{C} \Delta_{\zeta_{\mathfrak{g}_w}}^{\nu/2}$$

$${}^z I_\nu = {}^z \Delta_{\overline{z}}^{-\nu/2}$$

$$T_{\varrho, \nu}^{\mathbb{R}} = T_J^{\mathbb{C}} I_\nu$$

$$\mathbb{R} \overline{\Delta} \triangleleft_{\infty} \mathbb{C} \xleftarrow{\varrho_\nu} \mathbb{C} \overline{\Delta} \triangleleft_{\infty} \mathbb{C}$$

$$G_{\mathbb{R}} \text{ inv}$$

$$\mathcal{T}^{\mathbb{R}} = \varrho / I_\nu$$

$${}^x \mathcal{T}_\eta^{\mathbb{R}} = {}^x \Delta_x^{\nu/2} {}^x \eta = {}^x \Delta_x^\nu {}^x \eta$$

$$\int_{dx}^{\mathbb{R}\mathbb{Z}} x \Delta_x^{\nu-p} x \bar{\gamma}^x \overline{\varrho_\nu \mathbb{J}} = \int_{dz}^{\mathbb{C}\mathbb{Z}} z \Delta_z^{\nu-p} z \overline{\Delta_z^{-\nu}} z \overline{\gamma/I_\nu} z \mathbb{J}$$

$$z \bar{I}_\nu z I_\nu = z \overline{\Delta_z^{-\nu}}$$

$$\text{LHS} = \int_{dx}^{\mathbb{R}\mathbb{Z}} x \Delta_x^{-p} x \Delta_x^\nu x \bar{\gamma}^x \overline{\varrho_\nu \mathbb{J}} = \int_{dx}^{\mathbb{R}\mathbb{Z}} x \Delta_x^{-p} \overline{\mathcal{T}^{\mathbb{R}} \gamma}^x \overline{\varrho_\nu \mathbb{J}}$$

$$= \overline{\mathcal{T}^{\mathbb{R}} \gamma}^x \overline{\varrho_\nu \mathbb{J}} = \gamma \overline{\mathcal{T}^{\mathbb{R}} \varrho_\nu \mathbb{J}} = \gamma \overline{\mathcal{T}^{\mathbb{C}} I_\nu} = \gamma \overline{\mathbb{J} I_\nu} = \gamma / I_\nu \overline{\mathbb{J} I_\nu} = \text{RHS}$$

$$\mathbb{R}\mathbb{Z} \xleftarrow{\mathcal{P}_\nu} \mathbb{C}\mathbb{Z}: \quad x \overline{\mathcal{P}_\nu \mathbb{J}} = x \Delta_x^p \int_{dy}^{\overline{\pi}^{-1}(0)} y \Delta_y^\nu y \overline{\Delta_y^{-\nu}} \overline{\gamma(x:y)}^{y\gamma_x} \Delta_{y\gamma_x}^{-p} \mathbb{J}$$

$$\mathcal{P}_\nu \overline{\mathbb{J} \pi \times \mathbb{1}} = \overline{\mathcal{P}_\nu \mathbb{J} \mathbb{1}}$$

$$\overline{\mathbb{J} \pi \times \mathbb{1}} = y\gamma_x \mathbb{J} \overline{\pi \times \mathbb{1}} = y\gamma_x \mathbb{J} (y\gamma_x) \pi \mathbb{1} = y\gamma_x \mathbb{J}^x \mathbb{1}$$

$${}^x \text{LHS} = x \Delta_x^p \int_{dy}^{\overline{\pi}^{-1}(0)} y \Delta_y^\nu y \overline{\Delta_y^{-\nu}} \overline{\gamma(x:y)}^{y\gamma_x} \Delta_{y\gamma_x}^{-p} \overline{\pi \times \mathbb{1}} \mathbb{J}$$

$$= x \Delta_x^p \int_{dy}^{\overline{\pi}^{-1}(0)} y \Delta_y^\nu y \overline{\Delta_y^{-\nu}} \overline{\gamma(x:y)}^{y\gamma_x} \Delta_{y\gamma_x}^{-p} \mathbb{J}^x \mathbb{1} = {}^x \text{RHS}$$

$$\int_{dx}^{\mathbb{R}\overline{\mathbb{Z}}} {}^x\Delta_x^{-p} {}^x\overline{\mathcal{P}_\nu \mathcal{J}} = \int_{dz}^{\mathbb{C}\overline{\mathbb{Z}}} {}^z\Delta_z^{\nu-p} {}^z\overline{\Delta_{z^\#}^{-\nu}} {}^z\mathcal{J}$$

$$y^\# \gamma_x = y \gamma_x^\#$$

$$\text{LHS} = \int_{dx}^{\mathbb{R}\overline{\mathbb{Z}}} \int_{dy}^{\overline{\pi}^{-1}(0)} {}^y\Delta_y^\nu {}^y\overline{\Delta_{y^\#}^{-\nu}} \overline{\gamma(x:y)} {}^{y\gamma_x}\Delta_{y\gamma_x}^{-p} {}^{y\gamma_x}\mathcal{J} \stackrel{\text{inv}}{=} \int_{dx}^{\mathbb{R}\overline{\mathbb{Z}}} \int_{dy}^{\overline{\pi}^{-1}(0)} \overline{\gamma(x:y)} {}^{y\gamma_x}\Delta_{y\gamma_x}^{\nu-p} {}^{y\gamma_x}\overline{\Delta_{y\gamma_x^\#}^{-\nu}} {}^{y\gamma_x}\mathcal{J} = \text{RHS}$$

$$\mathbb{R}\overline{\mathbb{Z}} \triangleleft_{\infty} \mathbb{C} \xleftarrow[G_{\mathbb{R}} \text{ inv}]{\mathcal{P}_\nu} \mathbb{C}\overline{\mathbb{Z}} \triangleleft_{\infty} \mathbb{C}: \quad \mathcal{P}_\nu \overline{g \times \mathcal{J}} = \overline{g \times \mathcal{P}_\nu \mathcal{J}}$$

$$\begin{aligned} & \int_{dx}^{\mathbb{R}\overline{\mathbb{Z}}} {}^x\Delta_x^{-p} {}^x\overline{g \times \mathbf{1}} {}^x\overline{\mathcal{P}_\nu g \times \mathcal{J}} = \int_{dx}^{\mathbb{R}\overline{\mathbb{Z}}} {}^x\Delta_x^{-p} {}^x\overline{\mathcal{P}_\nu \overline{g \times \mathcal{J} \pi \times g \times \mathbf{1}}} = \int_{dx}^{\mathbb{R}\overline{\mathbb{Z}}} {}^x\Delta_x^{-p} {}^x\overline{\mathcal{P}_\nu \overline{g \times \mathcal{J} \pi \times \mathbf{1}}} \\ & = \int_{dz}^{\mathbb{C}\overline{\mathbb{Z}}} {}^z\Delta_z^{-p} {}^z\Delta_z^\nu {}^z\overline{\Delta_{\bar{z}}^{-\nu}} {}^z\overline{\mathcal{J} \pi \times \mathbf{1}} \stackrel{\text{inv}}{=} \int_{dz}^{\mathbb{C}\overline{\mathbb{Z}}} {}^z\Delta_z^{-p} {}^{zg}\Delta_{zg}^\nu {}^{zg}\overline{\Delta_{\bar{z}g}^{-\nu}} {}^{zg}\overline{\mathcal{J} \pi \times \mathbf{1}} = \int_{dw}^{\mathbb{C}\overline{\mathbb{Z}}} {}^w\Delta_w^{-p} {}^w\Delta_w^\nu {}^w\overline{\Delta_{\bar{w}}^{-\nu}} {}^w\overline{\mathcal{J} \pi \times \mathbf{1}} \\ & = \int_{dx}^{\mathbb{R}\overline{\mathbb{Z}}} {}^x\Delta_x^{-p} {}^x\overline{\mathcal{P}_\nu \overline{\mathcal{J} \pi \times \mathbf{1}}} = \int_{dx}^{\mathbb{R}\overline{\mathbb{Z}}} {}^x\Delta_x^{-p} {}^x\overline{\mathbf{1} \times \mathcal{P}_\nu \mathcal{J}} = \int_{dx}^{\mathbb{R}\overline{\mathbb{Z}}} {}^x\Delta_x^{-p} {}^{xg}\Delta_{xg}^\nu {}^{xg}\overline{\mathcal{P}_\nu \mathcal{J}} = \int_{dx}^{\mathbb{R}\overline{\mathbb{Z}}} {}^x\Delta_x^{-p} {}^x\overline{g \times \mathbf{1}} {}^x\overline{g \times \mathcal{P}_\nu \mathcal{J}} \end{aligned}$$

$$\int_{dx}^{\mathbb{R}\overline{\mathbb{Z}}} {}^x\Delta_x^{\nu-p} {}^x\overline{\varrho_\nu \mathcal{J}} = \int_{dx}^{\mathbb{R}\overline{\mathbb{Z}}} {}^x\Delta_x^{-p} {}^x\overline{\mathcal{P}_\nu \overline{\varrho_\nu \mathcal{J}}}$$

$$\text{LHS} = \int_{dz}^{\mathbb{C}\overline{\mathbb{Z}}} {}^z\Delta_z^{\nu-p} {}^z\overline{\Delta_{\bar{z}}^{-\nu}} {}^z\overline{\varrho_\nu \mathcal{J}} = \text{RHS}$$