

$$\underline{\mathfrak{h}} \triangleleft_{\infty} \mathfrak{h} \ni \mathfrak{b} \Rightarrow \mathfrak{h} \triangleleft_{\infty} \mathbb{K} \xleftarrow[\text{lin}]{\mathfrak{b} \times} \mathfrak{h} \triangleleft_{\infty} \mathbb{K}$$

$$\mathfrak{b} \times \underbrace{\gamma \dot{\gamma}} = \underbrace{\mathfrak{b} \times \gamma} \dot{\gamma} + \gamma \underbrace{\mathfrak{b} \times \dot{\gamma}}$$

$$\bigwedge_{\mathfrak{h} \in \mathfrak{h}} \mathfrak{b}_{\mathfrak{h}} \in \underline{\mathfrak{h}} \times \mathfrak{h}$$

$$\mathbb{K} \xleftarrow[\text{der}]{\mathfrak{b}_{\mathfrak{h}}} \mathfrak{h} \triangleleft_{\infty} \mathbb{K}$$

$${}^{\mathfrak{h}}(\mathfrak{b} \times \gamma) = \mathfrak{b}_{\mathfrak{h}} \dot{\gamma}_{\mathfrak{h}} = \mathfrak{b}_{\mathfrak{h}} {}^{\mathfrak{h}}\dot{\gamma} \in \mathbb{K} \times \gamma_{\mathfrak{h}} = \mathbb{K}$$

$$\overbrace{\mathfrak{b} \times \gamma \dot{\gamma}}^{\mathfrak{h}} = \mathfrak{b}_{\mathfrak{h}} \underbrace{\gamma \dot{\gamma}}_{\sim_{\mathfrak{h}}} = \underbrace{\mathfrak{b}_{\mathfrak{h}} \dot{\gamma}_{\mathfrak{h}}}_{\mathfrak{h}} + {}^{\mathfrak{h}}\gamma \underbrace{\mathfrak{b}_{\mathfrak{h}} \dot{\gamma}_{\mathfrak{h}}}_{\mathfrak{h}} = \overbrace{\mathfrak{b} \times \gamma}^{\mathfrak{h}} \dot{\gamma}_{\mathfrak{h}} + {}^{\mathfrak{h}}\gamma \overbrace{\mathfrak{b} \times \dot{\gamma}}^{\mathfrak{h}} \Rightarrow \mathfrak{b} \times \underbrace{\gamma \dot{\gamma}} = \underbrace{\mathfrak{b} \times \gamma} \dot{\gamma} + \gamma \underbrace{\mathfrak{b} \times \dot{\gamma}} \text{ on } \mathfrak{h}$$

$$\begin{array}{ccc} \mathfrak{h} \in \mathfrak{h} & \xrightarrow{\mathfrak{b}} & \mathfrak{h} : \mathfrak{b}_{\mathfrak{h}} \in \underline{\mathfrak{h}} \times \mathfrak{h} \\ \downarrow \mathfrak{b} & & \downarrow \mathfrak{s} \\ \mathbb{K} \ni \mathfrak{b}_{\mathfrak{h}} {}^{\mathfrak{h}}\dot{\gamma} & \xrightarrow{\gamma} & \mathbb{K} \times \mathbb{K} = \mathbb{K} \times \mathbb{K} \ni {}^{\mathfrak{h}}\gamma : \mathfrak{b}_{\mathfrak{h}} {}^{\mathfrak{h}}\dot{\gamma} \\ & & \Rightarrow \mathfrak{b} \times \gamma \in \mathfrak{h} \triangleleft_{\infty} \mathbb{K} \end{array}$$

$$\mathfrak{b} \in \mathfrak{h}_{\infty} \triangleright \mathfrak{h} \Rightarrow \begin{cases} \mathfrak{b} \times \mathfrak{b} = \mathfrak{b} \times \mathfrak{b} \\ \mathfrak{b} \times \mathfrak{b} \gamma - \mathfrak{b} \times \mathfrak{b} \gamma = \mathfrak{b} \times \mathfrak{b} \gamma \quad \wedge \gamma \in \mathfrak{h}_{\infty} \mathbb{K} \end{cases}$$

$$\text{Karte } U: \mathfrak{g} \Rightarrow \mathfrak{b}_U = \sum_i \mathfrak{b} \times \gamma^i \frac{\partial}{\partial \gamma^i}: \mathfrak{b} \times \gamma^i \in U \mathfrak{h}_{\infty} \mathbb{K}$$

$$\gamma \in \mathfrak{h}_{\infty} \mathbb{K} \Rightarrow \frac{\partial^2 \gamma}{\partial \gamma^i \partial \gamma^j} = \frac{\partial^2 \gamma}{\partial \gamma^j \partial \gamma^i}$$

$$\Rightarrow U(\mathfrak{b} \times \mathfrak{b} \gamma - \mathfrak{b} \times \mathfrak{b} \gamma) = \sum_{ij} \mathfrak{b} \times \gamma^i \frac{\partial}{\partial \gamma^i} \mathfrak{b} \times \gamma^j \frac{\partial \gamma}{\partial \gamma^j} - \mathfrak{b} \times \gamma^j \frac{\partial}{\partial \gamma^j} \mathfrak{b} \times \gamma^i \frac{\partial \gamma}{\partial \gamma^i}$$

$$= \sum_{ij} \mathfrak{b} \times \gamma^i \frac{\partial \mathfrak{b} \times \gamma^j}{\partial \gamma^i} \frac{\partial \gamma}{\partial \gamma^j} + \mathfrak{b} \times \gamma^i \mathfrak{b} \times \gamma^j \frac{\partial^2 \gamma}{\partial \gamma^i \partial \gamma^j} - \mathfrak{b} \times \gamma^j \frac{\partial \mathfrak{b} \times \gamma^i}{\partial \gamma^j} \frac{\partial \gamma}{\partial \gamma^i} - \mathfrak{b} \times \gamma^j \mathfrak{b} \times \gamma^i \frac{\partial^2 \gamma}{\partial \gamma^j \partial \gamma^i}$$

$$= \sum_{ij} \mathfrak{b} \times \gamma^i \frac{\partial \mathfrak{b} \times \gamma^j}{\partial \gamma^i} - \mathfrak{b} \times \gamma^j \frac{\partial \mathfrak{b} \times \gamma^i}{\partial \gamma^j} \frac{\partial \gamma}{\partial \gamma^j} = U(\mathfrak{b} \times \mathfrak{b} \gamma)$$

$$\wedge \delta \in \text{Der} | \mathfrak{h}_{\infty} \mathbb{K} = \frac{\mathfrak{h}_{\infty} \mathbb{K} \xleftarrow{\delta} \mathfrak{h}_{\infty} \mathbb{K}}{\delta \gamma^i = \delta \gamma^i + \gamma^i \delta^i} \Rightarrow \begin{cases} \text{eind} \\ \bigvee \mathfrak{b} \in \mathfrak{h}_{\infty} \mathfrak{h} \\ \mathfrak{b} \times \gamma = \delta \gamma \quad \wedge \gamma \in \mathfrak{h}_{\infty} \mathbb{K} \end{cases}$$

$$\text{Trg } \gamma = \frac{\mathfrak{h} \in \mathfrak{h}^-}{\mathfrak{h} \gamma \neq 0} \subset \mathfrak{h}$$

Trg $\delta\gamma \subset$ Trg γ supp-decr

$$\begin{aligned}
 \mathfrak{h} \in \mathfrak{h}\text{-L Trg } \gamma &\Rightarrow \bigvee_{U:\mathfrak{V}}^{\text{Karte}} \mathfrak{h}\text{-L Trg } \supset U \ni \mathfrak{h} \Rightarrow \bigvee^{\text{bump}} \left\{ \begin{array}{l} U \xrightarrow[\text{smooth}]{\mathfrak{V} \xrightarrow{\chi}} 0|1 \\ \mathfrak{h}|\mathfrak{V} \chi = 1 \end{array} \right. \\
 \text{cpt Trg } \chi \subset U \xrightarrow{\mathfrak{V}} &\Rightarrow \left\{ \begin{array}{l} U \xrightarrow[\text{smooth}]{\mathfrak{V} \times \chi} 0|1 \\ \mathfrak{h}(\mathfrak{V} \times \chi) = 1 \end{array} \right. \\
 \text{cpt Trg } \mathfrak{V} \times \chi \subset U &\Rightarrow \left\{ \begin{array}{l} \mathfrak{h} \xrightarrow[\text{smooth}]{\overline{\mathfrak{V} \times \chi} \text{ triv ext}} 0|1 \\ \mathfrak{h}\overline{\mathfrak{V} \times \chi} = 1 \end{array} \right. \\
 \text{cpt Trg } \overline{\mathfrak{V} \times \chi} \subset U &\Rightarrow \overline{\gamma \mathfrak{V} \times \chi} = 0 \Leftarrow U|\gamma = 0 \\
 \mathfrak{h}\text{-L}U|\overline{\mathfrak{V} \times \chi} = 0 &\Rightarrow 0 = \mathfrak{h}\overline{\delta\gamma \mathfrak{V} \times \chi} = \mathfrak{h}\overline{\delta\gamma} \mathfrak{h}\overline{\mathfrak{V} \times \chi} + \mathfrak{h}\gamma \mathfrak{h}\overline{\delta\mathfrak{V} \times \chi} = \mathfrak{h}\overline{\delta\gamma} \\
 &\Rightarrow \mathfrak{h}\text{-L Trg } \gamma|\delta\gamma = 0 \Rightarrow \text{Trg } \delta\gamma \subset \text{Trg } \gamma
 \end{aligned}$$

$$\mathfrak{J}_h \in \mathfrak{h}_{\infty} \mathbb{K} \xleftarrow[\text{surj}]{\text{germ}} \mathfrak{h}_{\infty} \mathbb{K} \ni \gamma$$

$$1 \in \mathfrak{h}_{\infty} \mathbb{K} \Rightarrow \bigvee \left\{ \begin{array}{l} h \in U \subset \mathfrak{h} \\ \gamma \in U \mathfrak{h}_{\infty} \mathbb{K} \end{array} \right. \quad \mathfrak{J}_h = 1 \xrightarrow{\text{OE}} U: \mathfrak{J} \text{ Karte} \Rightarrow \bigvee h \in V \underset{\text{off}}{\subset} U \Rightarrow {}^V \mathfrak{J} \underset{\text{off}}{\subset} {}^U \mathfrak{J}$$

$$\Rightarrow \bigvee^{\text{bump}} \left\{ \begin{array}{l} {}^U \mathfrak{J} \xrightarrow{\chi} 0|1 \\ \text{smooth} \\ {}^V \mathfrak{J} \chi = 1 \end{array} \right.$$

$$\text{cpt Trg } \chi \subset {}^U \mathfrak{J} \Rightarrow \left\{ \begin{array}{l} U \xrightarrow{\mathfrak{J} \times \chi} 0|1 \\ \text{smooth} \\ {}^V (\mathfrak{J} \times \chi) = 1 \end{array} \right.$$

$$\text{cpt Trg } \mathfrak{J} \times \chi \subset U \Rightarrow \left\{ \begin{array}{l} \overline{\mathfrak{J} \times \chi} \text{ triv forts} \\ \mathfrak{h} \xrightarrow{\text{smooth}} 0|1 \\ {}^V \overline{\mathfrak{J} \times \chi} = 1 \end{array} \right.$$

$$\text{cpt Trg } \overline{\mathfrak{J} \times \chi} \subset U \Rightarrow \mathfrak{J} \overline{\mathfrak{J} \times \chi} \in \mathfrak{h}_{\infty} \mathbb{K}$$

$${}^V (\mathfrak{J} \overline{\mathfrak{J} \times \chi}) = {}^V \mathfrak{J} \Rightarrow \left(\mathfrak{J} \overline{\mathfrak{J} \times \chi} \right)_h \underset{\sim}{=} \mathfrak{J}_h = \downarrow$$

$$\mathfrak{b}_h \downarrow = {}^h(\delta\gamma) \begin{cases} \downarrow = \mathfrak{q}_h \\ \gamma \in {}^h_{\infty}\mathbb{K} \end{cases} \Rightarrow \text{well-def } \mathfrak{b}_h \in \underline{\mathfrak{h}}_h$$

$$\begin{cases} \dot{\gamma} \in {}^h_{\infty}\mathbb{K} \\ \mathfrak{q}_h = \dot{\mathfrak{q}}_h \end{cases} \Rightarrow \bigvee_{h \in U \subset \mathfrak{h}} U|\gamma = U|\dot{\gamma} \Rightarrow U|\underline{\gamma - \dot{\gamma}} = 0 \Rightarrow \text{Trg } \underline{\gamma - \dot{\gamma}} \subset \mathfrak{h} \perp U \xrightarrow{1}$$

$$\text{Trg } \delta \underline{\gamma - \dot{\gamma}} \subset \mathfrak{h} \perp U \Rightarrow 0 = \overline{\delta \underline{\gamma - \dot{\gamma}}} = \overline{\delta \gamma - \delta \dot{\gamma}} = {}^h\delta\gamma - {}^h\delta\dot{\gamma} \Rightarrow {}^h\delta\gamma = {}^h\delta\dot{\gamma}$$

$$\mathfrak{b}_h \in \underline{\mathfrak{h}} \times \mathfrak{h} : \downarrow \in {}^h_{\infty}\mathbb{K} \xrightarrow{\text{OE}} \bigvee \dot{\gamma} \in {}^h_{\infty}\mathbb{K} \downarrow = \dot{\mathfrak{q}}_h$$

$$\mathfrak{b}_h \downarrow \downarrow = \mathfrak{b}_h \overline{\underline{\dot{\gamma}}_h} = \overline{{}^h\delta\dot{\gamma}} = \overline{{}^h\delta(\gamma - \dot{\gamma})} = \overline{{}^h\delta\gamma - {}^h\delta\dot{\gamma}} = \overline{{}^h\delta\gamma} - \overline{{}^h\delta\dot{\gamma}} = \mathfrak{b}_h \downarrow \downarrow \dot{\gamma} + {}^h\gamma \mathfrak{b}_h \downarrow$$

$$\bigwedge_{\gamma \in {}^h_{\infty}\mathbb{K}} \delta\gamma = \mathfrak{b} \times \gamma \leftarrow {}^h\delta\gamma = \mathfrak{b}_h \mathfrak{q}_h = \overline{\mathfrak{b} \times \gamma}$$

$$\mathfrak{h} \xrightarrow[\text{glatt}]{\mathfrak{b}} \underline{\mathfrak{h}} \times \mathfrak{h}$$

$$U: \mathcal{V} \text{ Karte} \Rightarrow \bigvee \mathfrak{h} \in V \underset{\text{off}}{\subseteq} U \begin{cases} U \xrightarrow[\text{smooth}]{\chi} 0|1 \\ \mathfrak{V}\chi = 1 \end{cases}$$

$$\text{cpt Trg } \chi \subset U \Rightarrow \begin{cases} \mathfrak{h} \xrightarrow[\text{smooth}]{\widehat{\chi} \text{ triv ext}} 0|1 \\ \mathfrak{V}\widehat{\chi} = 1 \end{cases} \quad \text{cpt Trg } \widehat{\chi} \subset U$$

$$\mathfrak{h} \supset \frac{\mathcal{V} = \mathcal{V}^1 \dots \mathcal{V}^n}{\text{glatt}} \rightarrow \mathbb{R}^n \Rightarrow \mathcal{V}^i \in U \underset{\infty}{\mathbb{K}} \Rightarrow \begin{cases} \widehat{\chi} \mathcal{V}^i \in {}^h_{\infty}\mathbb{K} \\ \widehat{\chi}^{\mathfrak{V}} \mathcal{V}^i = \mathfrak{V} \mathcal{V}^i \end{cases}$$

$$\Rightarrow \mathfrak{b}_V = \sum_{1 \leq i \leq n} \mathfrak{V} \left(\delta \widehat{\chi} \mathcal{V}^i \right) \frac{\partial}{\partial \mathcal{V}^i} = \sum_{1 \leq i \leq n} \mathfrak{V} \left(\delta \widehat{\chi} \mathcal{V}^i \right) \mathfrak{1}_i \Rightarrow V \xrightarrow[\text{glatt}]{\mathfrak{b}_V} \underline{V} \times V \Rightarrow \mathfrak{h} \xrightarrow[\text{glatt}]{\mathfrak{b}} \underline{\mathfrak{h}} \times \mathfrak{h}$$