$$ {}^{t=s}\partial_t\, \mathcal{E}^{t\flat} \ltimes \mathfrak{y}_t = \mathcal{E}^s_\flat \ltimes \underbrace{\dot{\mathfrak{y}}_s + \mathbb{b} \bowtie \mathfrak{y}_s} $$

$$ \frac{{}^{\mathsf{h}\mathcal{E}^{t\flat}}\mathfrak{y}_t - {}^{\mathsf{h}\mathcal{E}^s_\flat}\mathfrak{y}_s}{t-s} = \frac{{}^{\mathsf{h}\mathcal{E}^s_\flat}\mathfrak{y}_t - {}^{\mathsf{h}\mathcal{E}^s_\flat}\mathfrak{y}_s}{t-s} + \frac{{}^{\mathsf{h}\mathcal{E}^{t\flat}}\mathfrak{y}_t - {}^{\mathsf{h}\mathcal{E}^{t\flat}}\mathfrak{y}_s}{t-s} = \frac{{}^{\mathsf{h}\mathcal{E}^s_\flat}\mathfrak{y}_t - {}^{\mathsf{h}\mathcal{E}^s_\flat}\mathfrak{y}_s}{t-s} + \frac{\mathsf{h}\mathcal{E}^{t\flat} - \mathsf{h}\mathcal{E}^s_\flat}{t-s} \int_{d\vartheta}^{0|1} {}^{\mathsf{h}\mathcal{E}^s_\flat + \vartheta \underbrace{\mathsf{h}\mathcal{E}^{t\flat} - \mathsf{h}\mathcal{E}^s_\flat}}\underline{\mathfrak{y}}_t $$

$$ \Longrightarrow {}^{\mathsf{h}\mathcal{E}^s_\flat}\dot{\mathfrak{y}}_s + {}^{\mathsf{h}\mathcal{E}^s_\flat}\mathbb{b}\; {}^{\mathsf{h}\mathcal{E}^s_\flat}\underline{\mathfrak{y}}_s = {}^{\mathsf{h}\mathcal{E}^s_\flat}\dot{\mathfrak{y}}_s + {}^{\mathsf{h}\mathcal{E}^s_\flat}\underbrace{\mathbb{b} \bowtie \mathfrak{y}_s} $$

$$ {}^{t=s}\partial_t \underbrace{\mathcal{E}^{t\,{}_1\mathfrak{a}} \cdots \mathcal{E}^{t\,{}_n\mathfrak{a}}} \ltimes \mathfrak{y}_t = \underbrace{\mathcal{E}^{s\,{}_1\mathfrak{a}} \cdots \mathcal{E}^{s\,{}_n\mathfrak{a}}} \ltimes \dot{\mathfrak{y}}_s + \sum_{1 \leqslant k \leqslant n} \underbrace{\mathcal{E}^{s\,{}_1\mathfrak{a}} \cdots \mathcal{E}^{s\,{}_n\mathfrak{a}}} \ltimes \underbrace{{}_k\mathfrak{a} \bowtie \overbrace{\mathcal{E}^{s\,{}_{k+1}\mathfrak{a}} \cdots \mathcal{E}^{s\,{}_n\mathfrak{a}}} \ltimes \mathfrak{y}_s} $$

$n = 0$ klar

$$ 0 \leqslant n \curvearrowright n+1: \quad {}^{t=s}\partial_t \underbrace{\mathcal{E}^{t\,{}_0\mathfrak{a}} \cdots \mathcal{E}^{t\,{}_n\mathfrak{a}}} \ltimes \mathfrak{y}_t = {}^{t=s}\partial_t\, \mathcal{E}^{t\,{}_0\mathfrak{a}} \ltimes \underbrace{\overbrace{\mathcal{E}^{t\,{}_1\mathfrak{a}} \cdots \mathcal{E}^{t\,{}_n\mathfrak{a}}} \ltimes \mathfrak{y}_t} $$

$$ = \mathcal{E}^{s\,{}_0\mathfrak{a}} \ltimes {}^{t=s}\partial_t \underbrace{\mathcal{E}^{t\,{}_1\mathfrak{a}} \cdots \mathcal{E}^{t\,{}_n\mathfrak{a}}} \ltimes \mathfrak{y}_t + \mathcal{E}^{s\,{}_0\mathfrak{a}} \ltimes \underbrace{{}_0\mathfrak{a} \bowtie \overbrace{\mathcal{E}^{s\,{}_1\mathfrak{a}} \cdots \mathcal{E}^{s\,{}_n\mathfrak{a}}} \ltimes \mathfrak{y}_s} $$

$$ = \mathcal{E}^{s\,{}_0\mathfrak{a}} \ltimes \underbrace{\overbrace{\mathcal{E}^{s\,{}_1\mathfrak{a}} \cdots \mathcal{E}^{s\,{}_n\mathfrak{a}}} \ltimes \dot{\mathfrak{y}}_s} + \mathcal{E}^{s\,{}_0\mathfrak{a}} \ltimes \underbrace{{}_0\mathfrak{a} \bowtie \overbrace{\mathcal{E}^{s\,{}_1\mathfrak{a}} \cdots \mathcal{E}^{s\,{}_n\mathfrak{a}}} \ltimes \mathfrak{y}_s} $$

$$ + \mathcal{E}^{s\,{}_0\mathfrak{a}} \ltimes \sum_{1 \leqslant k \leqslant n} \underbrace{\mathcal{E}^{s\,{}_1\mathfrak{a}} \cdots \mathcal{E}^{s\,{}_k\mathfrak{a}} \ltimes \underbrace{{}_k\mathfrak{a} \bowtie \overbrace{\mathcal{E}^{s\,{}_{k+1}\mathfrak{a}} \cdots \mathcal{E}^{s\,{}_n\mathfrak{a}}} \ltimes \mathfrak{y}_s}} $$

$$ = \underbrace{\mathcal{E}^{s\,{}_0\mathfrak{a}} \cdots \mathcal{E}^{s\,{}_n\mathfrak{a}}} \ltimes \dot{\mathfrak{y}}_s + \sum_{0 \leqslant k \leqslant n} \underbrace{\mathcal{E}^{s\,{}_0\mathfrak{a}} \cdots \mathcal{E}^{s\,{}_n\mathfrak{a}}} \ltimes \underbrace{{}_k\mathfrak{a} \bowtie \overbrace{\mathcal{E}^{s\,{}_{k+1}\mathfrak{a}} \cdots \mathcal{E}^{s\,{}_n\mathfrak{a}}} \ltimes \mathfrak{y}_s} $$

$$ {}^{t=0}\partial_t \underbrace{\mathcal{E}^{t\,{}_1\mathfrak{a}} \cdots \mathcal{E}^{t\,{}_n\mathfrak{a}}} \ltimes \mathfrak{y} = \sum_{1 \leqslant k \leqslant n} {}_k\mathfrak{a} \bowtie \mathfrak{y} $$

$$ \partial_t \underbrace{\mathcal{E}^{t\flat}}\underbrace{\mathcal{E}^{t\natural}}\underbrace{\mathcal{E}^{-t\flat}}\underbrace{\mathcal{E}^{-t\natural}} \ltimes \mathfrak{y} $$

$$ = \mathcal{E}^{t\flat} \ltimes \underbrace{\mathbb{b} \bowtie \overbrace{\mathcal{E}^{t\natural}\mathcal{E}^{-t\flat}\mathcal{E}^{t\natural}} \ltimes \mathfrak{y}} + \underbrace{\mathcal{E}^{t\flat}\mathcal{E}^{t\natural}} \ltimes \underbrace{\mathbb{t} \bowtie \underbrace{\mathcal{E}^{-t\flat}\mathcal{E}^{-t\natural}} \ltimes \mathfrak{y}} $$

$$ - \underbrace{\mathcal{E}^{t\flat}\mathcal{E}^{t\natural}\mathcal{E}^{-t\flat}} \ltimes \underbrace{\mathbb{b} \bowtie \overbrace{\mathcal{E}^{-t\natural} \ltimes \mathfrak{y}}} - \underbrace{\mathcal{E}^{t\flat}\mathcal{E}^{t\natural}\mathcal{E}^{-t\flat}\mathcal{E}^{-t\natural}} \ltimes \underbrace{\mathbb{t} \bowtie \mathfrak{y}} $$

$$ {}^{t=0}\partial_t \underbrace{\mathcal{E}^{t\flat}\mathcal{E}^{t\natural}\mathcal{E}^{-t\flat}\mathcal{E}^{-t\natural}} \ltimes \mathfrak{y} = 0 $$

$$ \text{LHS} = \mathbb{b} \bowtie \mathfrak{y} + \mathbb{t} \bowtie \mathfrak{y} - \mathbb{b} \bowtie \mathfrak{y} - \mathbb{t} \bowtie \mathfrak{y} = 0 $$

$$ {}^{t=0}\partial_t \big| \mathcal{E}^{t_1 \mathfrak{b}} \cdots \mathcal{E}^{t_m \mathfrak{b}} \ltimes \mathbb{b} \rtimes \underbrace{\overbrace{\mathcal{E}^{t_{m+1} \mathfrak{b}} \cdots \mathcal{E}^{t_n \mathfrak{b}}} \ltimes \daleth} = \sum_{i \leqslant m} {}_i\mathfrak{b} \rtimes \underbrace{\mathbb{b} \rtimes \daleth} + \sum_{j > m} \mathbb{b} \rtimes \underbrace{{}_j\mathfrak{b} \rtimes \daleth} $$

$$ \text{LHS} = \sum_{i \leqslant m} {}_i\mathfrak{b} \rtimes \underbrace{\mathbb{b} \rtimes \daleth} + {}^{t=0}\partial_t \mathbb{b} \rtimes \underbrace{\overbrace{\mathcal{E}^{t_{m+1} \mathfrak{b}} \cdots \mathcal{E}^{t_n \mathfrak{b}}} \ltimes \daleth} = \sum_{i \leqslant m} {}_i\mathfrak{b} \rtimes \underbrace{\mathbb{b} \rtimes \daleth} + \mathbb{b} \rtimes {}^{t=0}\partial_t \underbrace{\overbrace{\mathcal{E}^{t_{m+1} \mathfrak{b}} \cdots \mathcal{E}^{t_n \mathfrak{b}}} \ltimes \daleth} = $$

$$ \sum_{i \leqslant m} {}_i\mathfrak{b} \rtimes \underbrace{\mathbb{b} \rtimes \daleth} + \mathbb{b} \rtimes \sum_{j > m} \underbrace{{}_j\mathfrak{b} \rtimes \daleth} = \text{RHS} $$

$$ {}^{t=0}\partial_t^2 \underbrace{\mathcal{E}^{t\mathbb{b}} \mathcal{E}^{t\mathbb{t}} \mathcal{E}^{-t\mathbb{b}} \mathcal{E}^{-t\mathbb{t}}} \ltimes \daleth = 2 \underbrace{\mathbb{b} \times \mathbb{t}} \rtimes \daleth $$

$$ \text{LHS} = {}^{t=0}\partial_t \mathcal{E}^{t\mathbb{b}} \ltimes \underbrace{\mathbb{b} \rtimes \overbrace{\underbrace{\mathcal{E}^{t\mathbb{t}} \mathcal{E}^{-t\mathbb{b}} \mathcal{E}^{t\mathbb{t}}} \ltimes \daleth}} + {}^{t=0}\partial_t \underbrace{\mathcal{E}^{t\mathbb{b}} \mathcal{E}^{t\mathbb{t}}} \ltimes \underbrace{\mathbb{t} \rtimes \overbrace{\mathcal{E}^{-t\mathbb{b}} \mathcal{E}^{-t\mathbb{t}}} \ltimes \daleth} $$

$$ - {}^{t=0}\partial_t \underbrace{\mathcal{E}^{t\mathbb{b}} \mathcal{E}^{t\mathbb{t}} \mathcal{E}^{-t\mathbb{b}}} \ltimes \underbrace{\mathbb{b} \rtimes \overbrace{\mathcal{E}^{-t\mathbb{t}} \ltimes \daleth}} - {}^{t=0}\partial_t \underbrace{\mathcal{E}^{t\mathbb{b}} \mathcal{E}^{t\mathbb{t}} \mathcal{E}^{-t\mathbb{b}} \mathcal{E}^{-t\mathbb{t}}} \ltimes \underbrace{\mathbb{t} \rtimes \daleth} $$

$$ = \mathbb{b} \rtimes \underbrace{\mathbb{b} \rtimes \daleth} + \mathbb{b} \rtimes \underbrace{\mathbb{t} \rtimes \daleth} - \mathbb{b} \rtimes \underbrace{\mathbb{b} \rtimes \daleth} - \mathbb{b} \rtimes \underbrace{\mathbb{t} \rtimes \daleth} + \mathbb{b} \rtimes \underbrace{\mathbb{t} \rtimes \daleth} + \mathbb{t} \rtimes \underbrace{\mathbb{t} \rtimes \daleth} - \mathbb{t} \rtimes \underbrace{\mathbb{b} \rtimes \daleth} - \mathbb{t} \rtimes \underbrace{\mathbb{t} \rtimes \daleth} $$

$$ - \mathbb{b} \rtimes \underbrace{\mathbb{b} \rtimes \daleth} - \mathbb{t} \rtimes \underbrace{\mathbb{b} \rtimes \daleth} + \mathbb{b} \rtimes \underbrace{\mathbb{b} \rtimes \daleth} + \mathbb{b} \rtimes \underbrace{\mathbb{t} \rtimes \daleth} - \mathbb{b} \rtimes \underbrace{\mathbb{t} \rtimes \daleth} - \mathbb{t} \rtimes \underbrace{\mathbb{t} \rtimes \daleth} + \mathbb{b} \rtimes \underbrace{\mathbb{t} \rtimes \daleth} + \mathbb{t} \rtimes \underbrace{\mathbb{t} \rtimes \daleth} $$

$$ = 2\mathbb{b} \rtimes \underbrace{\mathbb{t} \rtimes \daleth} - 2\mathbb{t} \rtimes \underbrace{\mathbb{b} \rtimes \daleth} = 2 \underbrace{\mathbb{b} \times \mathbb{t}} \rtimes \daleth $$

$$ \underbrace{\overbrace{\mathbb{b} \times \mathbb{t}} \mathcal{E}^{t}} \ltimes \daleth \backsim \mathcal{E}^{t\mathbb{b}/\sqrt{n}} \, \mathcal{E}^{t\mathbb{t}/\sqrt{n}} \, \mathcal{E}^{-t\mathbb{b}/\sqrt{n}} \, \mathcal{E}^{-t\mathbb{t}/\sqrt{n}} \ltimes \daleth $$

$$ \varphi(t) := \underbrace{\mathcal{E}^{t\mathbb{b}}} \underbrace{\mathcal{E}^{t\mathbb{t}}} \underbrace{\mathcal{E}^{-t\mathbb{b}}} \underbrace{\mathcal{E}^{-t\mathbb{t}}} \ltimes \daleth $$

$$ f(\varepsilon) = \varphi\left(\varepsilon^{1/2}\right) \Longrightarrow $$

$$ \partial_\varepsilon f(\varepsilon) = \frac{1}{2} \varepsilon^{-1/2} \, \partial_t \varphi\left(\varepsilon^{1/2}\right) $$

$$ \partial_\varepsilon^2 f(\varepsilon) = -\frac{1}{4} \varepsilon^{-3/2} \, \partial_t \varphi\left(\varepsilon^{1/2}\right) + \frac{1}{2} \varepsilon^{-1/2} \frac{1}{2} \varepsilon^{-1/2} \, \partial_t^2 \varphi\left(\varepsilon^{1/2}\right) $$

$$ \Longrightarrow 4\varepsilon \, \partial_\varepsilon^2 f(\varepsilon) + 2 \, \partial_\varepsilon f(\varepsilon) = \partial_t^2 \varphi\left(\varepsilon^{1/2}\right) \Longrightarrow \partial_\varepsilon f(0) = \frac{1}{2} \partial_t^2 \varphi(0) = \underbrace{\mathbb{b} \times \mathbb{t}} \rtimes \daleth $$