

$$S_1^e \xrightarrow{\text{circ}} S_\ell \xrightarrow{\pi} \mathfrak{S}_\ell$$