

$${}_{u_\ell}\mathfrak{S}^{\textcolor{brown}{O}}|_{\mathfrak{e}\mathbb{L}}=\mathfrak{S}_1^{\textcolor{brown}{O}}|_{\mathfrak{e}\mathbb{L}}\times {}_{u_\ell}\mathfrak{S}_{-}^{\textcolor{brown}{O}}|_{\mathfrak{e}\mathbb{L}}\times \sum_{\ell < i < j \leqslant r}\mathfrak{S}^{\textcolor{teal}{I}}_j-i|_{\mathfrak{e}\mathbb{L}}\times \sum_{i < j > \ell}\mathfrak{S}^{-}_j-i|_{\mathfrak{e}\mathbb{L}}\times \sum_{0 \leqslant i \leqslant j > \ell}\mathfrak{S}^{-}_j+i|_{\mathfrak{e}\mathbb{L}}$$

$${}^{u_\ell}\mathfrak{S}|_{\mathfrak{e}\mathbb{L}}={}_{u_\ell}\mathfrak{S}|_{\mathfrak{e}\mathbb{L}}\times {}^{u_\ell}\mathfrak{S}_{-}^{\textcolor{brown}{O}}|_{\mathfrak{e}\mathbb{L}}\times \sum_{1 \leqslant i < j \leqslant \ell}\mathfrak{S}^{\textcolor{teal}{I}}_j-i|_{\mathfrak{e}\mathbb{L}}\times \sum_{0 \leqslant i \leqslant j \leqslant \ell}\mathfrak{S}^{\textcolor{teal}{I}}_j+i|_{\mathfrak{e}\mathbb{L}}$$

$${}_{u_\ell}\mathfrak{S}_{-}^{\textcolor{brown}{O}}|_{\mathfrak{e}\mathbb{L}}=\mathbb{K}\frac{2\overset{*}{e}_ke_k}{k>\ell}$$

$${}^{u_\ell}\mathfrak{S}_{-}^{\textcolor{brown}{O}}|_{\mathfrak{e}\mathbb{L}}=\mathbb{K}\frac{2\overset{*}{e}_ke_k}{k\leqslant \ell}$$

$$u_\ell\delta=\sum_{k\leqslant \ell}e_k\delta=0$$

$$u_\ell 2\overset{*}{e}_ke_k=2u_\ell\overset{*}{e}_ke_k=\begin{cases} 2e_k&k\leqslant \ell\\ 0&k>\ell\end{cases}$$

$$u_\ell\overset{*}{a}e_j=\begin{cases} a/2&i\leqslant \ell\\ 0&i>\ell\end{cases}$$

$$u_\ell\overset{*}{e}_ja=\begin{cases} a/2&j\leqslant \ell\\ 0&j>\ell\end{cases}$$

$$\underbrace{a\frac{\partial}{\partial z}}_{u_\ell}=a$$

$$\underbrace{z\overset{*}{a}z\frac{\partial}{\partial z}}_{u_\ell}=u_\ell\overset{*}{a}u_\ell=\begin{cases} -a/2&i< j\leqslant \ell\\ -a&i=j\leqslant \ell\\ 0&i\leqslant j>\ell\end{cases}$$

$$\underbrace{b+2z\overset{*}{b}e}_{u_\ell}=b+2u_\ell\overset{*}{b}e=b$$

$$\underbrace{z\overset{*}{b}z+2z\overset{*}{e}b}_{u_\ell}=u_\ell\overset{*}{b}u_\ell+2u_\ell\overset{*}{e}b=\begin{cases} b&j\leqslant \ell\\ 0&j>\ell\end{cases}$$

$${}_{u_{\ell^\infty}}\mathfrak{S}|_{\mathfrak{e}\mathbb{L}}=\mathfrak{S}_1^{\bullet}|_{\mathfrak{e}\mathbb{L}}\times {}_{u_\ell}\mathfrak{S}_{-}^{\bullet}|_{\mathfrak{e}\mathbb{L}}\times \sum_{\ell < i < j \leqslant r}\mathfrak{S}^{\textcolor{teal}{I}}_j-i|_{\mathfrak{e}\mathbb{L}}\times \sum_{i < j > \ell}\mathfrak{S}^{-}_j-i|_{\mathfrak{e}\mathbb{L}}$$

$${}^{u_\ell}\mathfrak{S}|_{\mathfrak{e}\mathbb{L}}={}_{u_{\ell^\infty}}\mathfrak{S}|_{\mathfrak{e}\mathbb{L}}\times {}^{u_\ell}\mathfrak{S}_{-}^{\bullet}|_{\mathfrak{e}\mathbb{L}}\times \sum_{1 \leqslant i < j \leqslant \ell}\mathfrak{S}^{\textcolor{teal}{I}}_j-i|_{\mathfrak{e}\mathbb{L}}\times \sum_{0 \leqslant i \leqslant j \leqslant \ell}\mathfrak{S}^{\textcolor{teal}{I}}_j+i|_{\mathfrak{e}\mathbb{L}}$$

$${}_{u_\ell}\mathfrak{S}^\bullet_- \mid {}_\mathfrak{e}\mathbb{L} = \mathbb{K} \frac{2\mathring{e}_k^* e_k}{k > \ell}$$

$${}^{u_\ell}\mathfrak{S}^\bullet_- \mid {}_\mathfrak{e}\mathbb{L} = \mathbb{K} \frac{2\mathring{e}_k^* e_k}{k \leqslant \ell}$$

$$u_\ell \delta = \sum_{k \leqslant \ell} e_k \delta = 0$$

$$u_\ell 2\mathring{e}_k^* e_k = 2u_\ell \mathring{e}_k^* e_k = \begin{cases} 2e_k & k \leqslant \ell \\ 0 & k > \ell \end{cases}$$

$$u_\ell \mathring{a} e_j = \begin{cases} a/2 & i \leqslant \ell \\ 0 & i > \ell \end{cases}$$

$$u_\ell \mathring{e}_j a = \begin{cases} a/2 & j \leqslant \ell \\ 0 & j > \ell \end{cases}$$

$$\underline{a \frac{\partial}{\partial z}}_{u_\ell} = a$$

$$\underline{z \mathring{a} z \frac{\partial}{\partial z}}_{u_\ell} = u_\ell \mathring{a} u_\ell = \begin{cases} -a/2 & i < j \leqslant \ell \\ -a & i = j \leqslant \ell \\ 0 & i \leqslant j > \ell \end{cases}$$

$$\underline{b + 2z \mathring{b} e}_{u_\ell} = b + 2u_\ell \mathring{b} e = b$$

$$\underline{z \mathring{b} z + 2z \mathring{e} b}_{u_\ell} = u_\ell \mathring{b} u_\ell + 2u_\ell \mathring{e} b = \begin{cases} b & j \leqslant \ell \\ 0 & j > \ell \end{cases} = b$$

$$\underline{z \mathring{b} z + 2z \mathring{e} b}_{u_\ell} = u_\ell \mathring{b} u_\ell + 2u_\ell \mathring{e} b = \begin{cases} b & j \leqslant \ell \\ 0 & j > \ell \end{cases}$$