

$$\int_{du}^{\mathbb{U}X^{\mathbb{C}}} u X_{\varkappa}^{\mathbb{C}} e^{-u} \bar{\Delta}^{\sigma} e^{-u} \Delta^{\tau} = \frac{\Gamma_{d/r}^X \Gamma_{\varkappa+\alpha+d/r}^X \Gamma_{\beta+d/r}^X}{\Gamma_{-\sigma}^X \Gamma_{\sigma+d/r}^X \Gamma_{\varkappa+\alpha+\beta+2d/r}^X}$$

$$\text{LHS} = \frac{\Gamma_{1+a/2}^r (2\pi)^{d-r}}{\Gamma_{1+ra/2} \Gamma_{-\sigma}^X \Gamma_{\sigma+d/r}^X} \int_{dx}^{0\mathbb{R}_1^r} x X_{\varkappa}^{\mathbb{C}} \prod_j x_j^{-\sigma-d/r} \overbrace{1-x_j}^{\sigma+\tau} \prod_{i<j} \overbrace{x_i-x_j}^a = \text{RHS}$$

$$\int_{du}^{\mathbb{U}X^{\mathbb{C}}} e^{-u} \Delta^{\sigma} e^{-u} \bar{\Delta}^{\tau} = \int_{dx}^X e^{-\overbrace{x-ie}^{-1} \overbrace{x+ie}^{-1}} \Delta^{\sigma} e^{-\overbrace{x-ie}^{-1} \overbrace{x+ie}^{-1}} \bar{\Delta}^{\tau} e+x^2 \Delta^{-d/r} = \int_{dx}^X 2i \overbrace{x+ie}^{-1} \Delta^{\sigma} 2i \overbrace{x+ie}^{-1} \bar{\Delta}^{\tau} e+x^2 \Delta^{-d/r}$$

$$= \int_{dx_i}^{\mathbb{R}^r} \prod_j \overbrace{x_j+i}^{-\sigma} \overbrace{x_j-i}^{-\tau} \overbrace{1+x_j^2}^{-d/r} \prod_{i<j} \overbrace{x_j-x_i}^a$$

$$e^{-z} \Delta^{\sigma} e^{-z} \bar{\Delta}^{\tau} = 2^{-\tau n} \prod_j \frac{\Gamma_{\sigma+\tau+j}}{\Gamma_{\sigma+j}} \sum_{\varkappa} \frac{\Gamma_{\varkappa-\sigma}^X}{\Gamma_{\varkappa+\tau+n}^X} d_{\varkappa} x X_{\varkappa}^{\mathbb{C}} = 2^{-\tau n} \prod_j \frac{\Gamma_{\sigma+\tau+j}}{\Gamma_{\sigma+j}} \sum_{\varkappa} \prod_j \frac{\Gamma_{\varkappa_j-j+1-\sigma}}{\Gamma_{\varkappa_j-j+1+\tau+n}} d_{\varkappa} x X_{\varkappa}^{\mathbb{C}}$$