



$$u\tilde{g} = \overbrace{u\tilde{g}k^{-1}} : u\tilde{g} = uk \text{ well-def}$$

$$u\tilde{g} = uh = uk \Rightarrow uhk^{-1} = u \Rightarrow \overbrace{u\tilde{g}k^{-1}} = \overbrace{u\tilde{g}h^{-1}} \overbrace{hk^{-1}}$$

$$\Rightarrow \overbrace{u\tilde{g}k^{-1}} = \overbrace{u\tilde{g}h^{-1}} \overbrace{hk^{-1}}^{\equiv 1} = \overbrace{u\tilde{g}h^{-1}}$$

$${}^u\tilde{g} \ {}^u\tilde{g}\tilde{y} = {}^u\tilde{g}\tilde{y} \text{ co-cycle}$$

$$\begin{cases} {}^u\tilde{g} = uk \\ {}^u\tilde{g}\tilde{y} = {}^u\tilde{g}h \end{cases} \Rightarrow {}^u\tilde{g}\tilde{y} = {}^u\tilde{g}\tilde{y} = {}^u\tilde{g}h = ukh$$

$$\Rightarrow {}^u\tilde{g}\tilde{y} \widehat{kh}^{-1} = {}^u\tilde{g} \ {}^u\tilde{g}\tilde{y} \ h^{-1} \ k^{-1} = {}^u\tilde{g} \ k^{-1} \ k \ {}^u\tilde{g}\tilde{y} \ h^{-1} \ k^{-1}$$

$$\Rightarrow \underbrace{{}^u\tilde{g}\tilde{y} \widehat{kh}^{-1}}_u = \underbrace{{}^u\tilde{g}k^{-1}}_u \underbrace{k \ {}^u\tilde{g}\tilde{y} \ h^{-1} \ k^{-1}}_u = \underbrace{{}^u\tilde{g}k^{-1}}_u \underbrace{{}^u\tilde{g}\tilde{y} \ h^{-1}}_{u\tilde{g}}$$

$$\begin{array}{ccccc} \mathbb{R} \times S^k & \xrightarrow{\hat{g}} & \mathbb{R} \times S^k & \xrightarrow{\hat{y}} & \mathbb{R} \times S^k \\ & & & \searrow & \nearrow \\ & & & \hat{g}\tilde{y} & \end{array}$$

$$\lambda: u \rtimes g = \lambda^{u\tilde{g}}: {}^u\tilde{g}$$

$$\overline{\lambda: u \rtimes g} \rtimes y = \overline{\lambda^{u\tilde{g}}: {}^u\tilde{g}} \rtimes y = \overline{\lambda^{u\tilde{g} \ {}^u\tilde{g}\tilde{y}}: {}^u\tilde{g}\tilde{y}} = \overline{{}^u\tilde{g}\tilde{y}}: {}^u\tilde{g}\tilde{y} = \lambda: u \rtimes gy$$