

$$K^\times = K \setminus (0) = \begin{cases} a \in K \\ a \neq 0 \end{cases}$$

$$\bigvee_{e \neq 0} a \cdot e \stackrel{\text{M0}}{=} a = e \cdot a \begin{cases} e \text{ eind} \\ (1) \xrightarrow{e} K^\times \end{cases}$$

$$a \cdot \tilde{e} = a \Rightarrow e \stackrel{\text{Vor}}{=} e \cdot \tilde{e} \stackrel{\text{2M}}{=} \tilde{e}$$

$$\bigwedge_a^{K^\times} \bigvee_{\tilde{a}}^{K^\times} a \cdot \tilde{a} \stackrel{\text{M1}}{=} e = \tilde{a}^{-1} \cdot a \begin{cases} \tilde{a}^{-1} \text{ eind} \\ K^\times \xrightarrow{(\tilde{a}^{-1})} K^\times \end{cases}$$

$$a \cdot \tilde{a} = 1 \Rightarrow \tilde{a} = \tilde{a} \cdot e = \tilde{a} \underbrace{a \tilde{a}^{-1}} = \tilde{a} \tilde{a} \tilde{a}^{-1} = e \cdot \tilde{a}^{-1} = \tilde{a}^{-1}$$

$$\tilde{a}^{-1} \in K^\times \wedge \overbrace{\tilde{a}^{-1}}^{-1} = a$$

$$\nmid \tilde{a}^{-1} = 0 \Rightarrow e = a \cdot \tilde{a}^{-1} = a \cdot 0 = 0 \nmid$$

$$\overbrace{\tilde{a}^{-1}}^{-1} = e \cdot \overbrace{\tilde{a}^{-1}}^{-1} = \overbrace{a \tilde{a}^{-1}} \cdot \overbrace{\tilde{a}^{-1}}^{-1} = a \cdot \underbrace{\tilde{a}^{-1} \cdot \overbrace{\tilde{a}^{-1}}^{-1}} = a \cdot e = a$$

$$\text{ganz / } \begin{cases} a \cdot b = 0 & \Rightarrow a = 0 \vee b = 0 \\ a \in K^\times \ni b & \Rightarrow a \cdot b \in K^\times \\ K^\times \times K^\times \xrightarrow{\text{mult}} K^\times \end{cases}$$

$$a \neq 0 \Rightarrow b = e \cdot b = \underbrace{\tilde{a}^{-1} a} b = \tilde{a}^{-1} \underbrace{a \cdot b} \stackrel{\text{Vor}}{=} \tilde{a}^{-1} 0 = 0$$

$$a \in K^\times \ni b \Rightarrow \overbrace{a \cdot b}^{-1} = \tilde{b} \tilde{a}^{-1}$$

$$\begin{cases} a \neq 0 \\ a \cdot b = a \cdot c \end{cases} \xrightarrow[\text{canc}]{\text{mult}} b = c$$

$$b \stackrel{0M}{=} e \cdot b \stackrel{-M}{=} \underbrace{a^{-1} \cdot a}_{2M} \cdot b \stackrel{-1}{=} a^{-1} \cdot \underbrace{a \cdot b}_{\text{Vor}} \stackrel{-1}{=} a^{-1} \cdot \underbrace{a \cdot c}_{2M} \stackrel{-1}{=} \underbrace{a^{-1} \cdot a}_{-M} \cdot c \stackrel{-M}{=} e \cdot c \stackrel{0M}{=} c$$