

$$R \xrightarrow[\text{abs value}]{\overline{()}} R_+$$

$$\overline{0} \stackrel{\text{refl}}{=} 0$$

$$\overline{x} = 0 \stackrel{\text{asym}}{\iff} x = 0$$

$$\overline{-a} \stackrel{\text{symm}}{=} \overline{a}$$

$$\overline{a+b} \stackrel{\text{trans}}{\leq} \overline{a} + \overline{b}$$

$$\overline{a \cdot b} = \overline{a} \overline{b}$$

$$a \geq 0 \leq b \Rightarrow a \cdot b \geq 0 \Rightarrow \overline{a \cdot b} = a \cdot b = \overline{a} \overline{b}$$

$$a \leq 0 \leq b \Rightarrow a \cdot b \leq 0 \Rightarrow \overline{a \cdot b} = -\underline{a \cdot b} = \underline{-a} b = \overline{a} \overline{b}$$

$$a \geq 0 \geq b \Rightarrow a \cdot b \leq 0 \Rightarrow \overline{a \cdot b} = -\underline{a \cdot b} = a \underline{-b} = \overline{a} \overline{b}$$

$$a \leq 0 \geq b \Rightarrow a \cdot b \geq 0 \Rightarrow \overline{a \cdot b} = a \cdot b = \underline{-a} \underline{-b} = \overline{a} \overline{b}$$