$$R\colon + : \cdot \ \text{field}$$

$$R\colon \leqslant \ \text{total 0rder}$$

$$x \leqslant y \underset{\text{OA}}{\Longrightarrow} x + z \leqslant y + z$$

$$a \leqslant b \Leftrightarrow 0 \leqslant b - a \Leftrightarrow b - a \geqslant 0$$

$$\Longrightarrow: \quad a \leqslant b \Longrightarrow 0 \underset{-A}{=} a + \underbrace{-a} \underset{\text{AP}}{\leqslant} b + (-a) = b - a$$

$$\Longleftarrow: \quad a \underset{0A}{=} 0 + a \overset{\text{AP}}{\underset{\text{V0r}}{\leqslant}} \underbrace{b - a} + a \underset{2A}{=} b + \underbrace{-a + a} \underset{-A}{=} b + 0 \underset{0A}{=} b$$

$$a \leqslant b \Longrightarrow -b \leqslant -a$$

$$-b \underset{0A}{=} 0 + \underbrace{-b} \underset{-A}{=} \overbrace{a + \underbrace{-a}} + \underbrace{-b} \underset{2A}{=} a + \overbrace{\underbrace{-a} + \underbrace{-b}} \underset{\text{AP}}{\leqslant} b + \overbrace{\underbrace{-a} + \underbrace{-b}}$$

$$\underset{1A}{=} b + \overbrace{\underbrace{-b} + \underbrace{-a}} \underset{2A}{=} \overbrace{b + \underbrace{-b}} + \underbrace{-a} \underset{-A}{=} 0 + \underbrace{-a} \underset{0A}{=} -a$$

$$a \leqslant 0 \leqslant b \Longrightarrow -b \leqslant 0 \leqslant -a$$

$$0 = -0$$

$$0 \leqslant x \wedge 0 \leqslant y \overset{\text{OM}}{\Longrightarrow} 0 \leqslant xy$$

$$a \leqslant b \wedge 0 \leqslant c \Longrightarrow a \cdot c \leqslant b \cdot c$$

$$a \leqslant b \underset{\text{AP1}}{\Longrightarrow} 0 \leqslant b - a \Longrightarrow 0 \underset{\text{MP}}{\leqslant} \underbrace{b - a} c = \overbrace{b + \underbrace{-a}} c \underset{\text{AM}}{=} b \cdot c + \underbrace{-a} \cdot c \underset{\text{M6}}{=} b \cdot c - a \cdot c \underset{\text{AP1}}{\Longrightarrow} a \cdot c \leqslant b \cdot c$$

$$a \leqslant b \wedge c \leqslant 0 \Longrightarrow a \cdot c \geqslant b \cdot c \Leftrightarrow c \cdot b \leqslant c \cdot a$$

$$-c \geqslant 0 \Longrightarrow c \cdot a - c \cdot b \underset{1A}{=} -c \cdot b + c \cdot a = -c \cdot b - \overbrace{- \underbrace{c \cdot a}} = -c \cdot b - \underbrace{-c} \cdot a \underset{\text{AM}}{=} \underbrace{-c}\,\underbrace{b - a} \underset{\text{MP}}{\geqslant} 0 \underset{\text{AP0}}{\Longrightarrow} c \cdot a \geqslant c \cdot b$$

$$0 \leqslant a^2$$

$$\begin{cases} 0 \leqslant a & \Longrightarrow 0 \underset{M0}{=} 0 \cdot a \underset{\text{MP}}{\leqslant} aa = a^2 \\ a \leqslant 0 & \underset{\text{AP2}}{\Longrightarrow} -a \geqslant 0 \underset{\text{MP}}{\Longrightarrow} 0 \leqslant \underbrace{-a}\,\underbrace{-a} \underset{M7}{=} a^2 \end{cases}$$

$$0 < e$$

$$0 \leqslant e^2 = e \wedge \cdot 0 \neq e$$

$$0 < a \Longrightarrow 0 < \overset{-1}{a}$$

$$\overset{-1}{a} \neq 0$$

$$\lightning\ 0 \geqslant \overset{-1}{a} \Longrightarrow 0 = a \cdot 0 \underset{\text{MP2}}{\geqslant} a\,a^{-1} = e\ \lightning$$

$$a < 0 \Longrightarrow \overset{-1}{a} < 0$$

$$a < 0 \Longrightarrow 0 < -a \underset{\text{MP5}}{\Longrightarrow} 0 < \overbrace{-a}^{-1} = -\overset{-1}{a} \underset{\text{AP1}}{\Longrightarrow} \overset{-1}{a} < 0$$

$$0 < a \leqslant b \Longrightarrow 0 < \overset{-1}{b} \leqslant \overset{-1}{a}$$

$$\overset{-1}{a} - \overset{-1}{b} = \underset{>0}{\overset{-1}{a}}\,\underbrace{b - a}_{\geqslant 0}\,\underset{>0}{\overset{-1}{b}} \underset{\text{MP}}{\geqslant} 0 \underset{\text{AP2}}{\Longrightarrow} \overset{-1}{b} \leqslant \overset{-1}{a}$$

$$a \geqslant -1 \underset{\text{Bern}}{\Longrightarrow} \overparen{1+a}^{\,n} \geqslant 1 + n \cdot a$$

$$\underline{0 \leqslant n \curvearrowright n+1}: \quad \overparen{1+a}^{\,n+1} = \underbrace{1+a}_{\geqslant 0} \cdot \overparen{1+a}^{\,n} \underset{\text{ind}}{\geqslant} \underbrace{1+a} \cdot \underbrace{1+n \cdot a} = 1 + \underbrace{n+1}a + \underbrace{n \cdot a^2}_{\geqslant 0} \geqslant 1 + \underbrace{n+1}a$$