$$x{:}y \in R{\times}R \xrightarrow[\text{metric}]{|} \underline{\angle R} \ni x|y = \overline{x-y}$$

$$\begin{cases} x|x = \overline{x-x} = \overline{0} = 0 & \text{refl} \\ 0 = x|y = \overline{x-y} \Longrightarrow 0 = x-y \Longrightarrow x = y & \text{asym} \\ y|x = \overline{y-x} = \overline{-\underline{x-y}} = \overline{x-y} = x|y & \text{symm} \\ x|y+y|z = \overline{x-y} + \overline{y-z} \geqslant \overline{\underline{x-y} + \underline{y-z}} = \overline{x-z} = x|z & \text{trans} \end{cases}$$

$$R^a_{\leqslant r} = \frac{x \in R}{x|a = \overline{x-a} \leqslant r}$$

$$R^a_{< r} = \frac{x \in R}{x|a = \overline{x-a} < r}$$

$$\underline{x+z}|\underline{y+z} \underset{\text{invar}}{=} x|y$$

$$\text{LHS} = \overline{\underline{x+z} - \underline{y+z}} = \overline{x-y} = \text{RHS}$$