$$R\colon + : \cdot : \leqslant \ \text{ordered field}$$

$$\underline{\angle R} = \frac{a \in R}{a \geqslant 0}$$

$$R \xrightarrow[\text{abs value}]{\ulcorner () \urcorner} \underline{\angle R}$$

$$0 \leqslant \ulcorner a \urcorner = \begin{cases} a & a \geqslant 0 \\ -a & a \leqslant 0 \end{cases} = \begin{cases} a & a > 0 \\ 0 & a = 0 \\ -a & a < 0 \end{cases}$$

$$-\ulcorner a \urcorner \leqslant a \leqslant \ulcorner a \urcorner$$

$$\begin{cases} a \geqslant 0 & \Longrightarrow \ulcorner a \urcorner = a \Longrightarrow -\ulcorner a \urcorner = -a \leqslant 0 \leqslant a = \ulcorner a \urcorner \\ a \leqslant 0 & \Longrightarrow \ulcorner a \urcorner = -a \Longrightarrow -\ulcorner a \urcorner = a \leqslant 0 \leqslant -a = \ulcorner a \urcorner \end{cases}$$

$$\ulcorner a \urcorner \leqslant c \Leftrightarrow -c \leqslant a \leqslant c$$

$$\Longrightarrow : \quad \ulcorner a \urcorner \leqslant c \Longrightarrow -c \leqslant -\ulcorner a \urcorner \leqslant a \leqslant \ulcorner a \urcorner \leqslant c$$

$$\Longleftarrow : \quad -c \leqslant a \leqslant c \Longrightarrow \begin{cases} a \geqslant 0 & \Longrightarrow \ulcorner a \urcorner = a \leqslant c \\ a \leqslant 0 & \Longrightarrow \ulcorner a \urcorner = -a \leqslant -\underbrace{-c} = c \end{cases}$$

$$\begin{cases} \ulcorner 0 \urcorner = 0 & \text{refl} \\ \ulcorner x \urcorner = 0 \Leftrightarrow x = 0 & \text{asym} \\ \ulcorner -a \urcorner = \ulcorner a \urcorner & \text{symm} \end{cases}$$

$$\ulcorner a + b \urcorner \underset{\text{trans}}{\leqslant} \ulcorner a \urcorner + \ulcorner b \urcorner$$

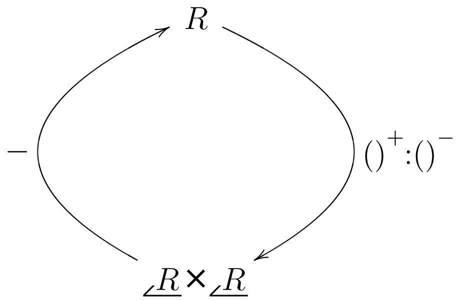
$$\begin{cases} a + b \geqslant 0 & \Longrightarrow \ulcorner a + b \urcorner = a + b \leqslant \ulcorner a \urcorner + \ulcorner b \urcorner \\ a + b \leqslant 0 & \Longrightarrow \ulcorner a + b \urcorner = -\underbrace{a + b} = \underbrace{-a} + \underbrace{-b} \leqslant \ulcorner a \urcorner + \ulcorner b \urcorner \end{cases}$$

$$\overline{\overline{a} - \overline{b}} \leqslant \overline{a - b}$$

$$\text{OE } \overline{a} \geqslant \overline{b} \Rightarrow \overline{\overline{a} - \overline{b}} = \overline{a} - \overline{b} = \overline{\underbrace{a - b} + b} - \overline{b} \underset{\text{trans}}{\leqslant} \underbrace{\overline{a - b} + \overline{b}} - \overline{b} = \overline{a - b}$$

$$\begin{cases} R \xrightarrow[\text{pos part}]{()^{+}} \underline{\angle R} & : \quad x^{+} = \dfrac{\overline{x} + x}{2} = \begin{cases} x & x \geqslant 0 \\ 0 & x \leqslant 0 \end{cases} \\ R \xrightarrow[\text{neg part}]{()^{-}} \underline{\angle R} & : \quad x^{-} = \dfrac{\overline{x} - x}{2} = \begin{cases} 0 & x \geqslant 0 \\ -x & x \leqslant 0 \end{cases} \end{cases}$$

$$\begin{cases} x = x^{+} - x^{-} & \text{add decomposition} \\ \overline{x} = x^{+} + x^{-} \end{cases}$$



$$\overbrace{a \underset{2}{+} b}^{n} \leqslant \overset{n}{a} \underset{2}{+} \overset{n}{b}$$

$$a \overset{n}{+} b \leqslant 2^{n-1} \underbrace{\overset{n}{a} + \overset{n}{b}}$$

$$a b^{n} + b a^{n} - a^{n+1} - b^{n+1} = \underbrace{a - b}\underbrace{b^{n} - a^{n}} \leqslant 0 \Rightarrow a^{n} b + b^{n} a \leqslant a^{n+1} + b^{n+1}$$

$$\Rightarrow \overbrace{a + b}^{n+1} = \overbrace{a + b}^{n} \underbrace{a + b} \underset{a + b \geqslant 0}{\overset{\text{ind}}{\leqslant}} 2^{n-1} \underbrace{a^{n} + b^{n}}\underbrace{a + b}$$

$$= 2^{n-1} \underbrace{a^{n+1} + b^{n+1} + a^{n} b + b^{n} a} \leqslant 2^{n-1} \underbrace{a^{n+1} + b^{n+1} + a^{n+1} + b^{n+1}} = 2^{n} \underbrace{a^{n} + b^{n}}$$