

$$o \in H = a|b \xrightarrow{\text{isoton}} \mathbb{R}$$

$$a < o: \overset{<}{\circ} \overset{\circ}{\underset{\cdot}{\vee}} = \bigwedge_{H \ni x < o} x \overset{\circ}{\underset{\cdot}{\vee}} \leq \overset{\circ}{\underset{\cdot}{\vee}}$$

$$o < b: \overset{>}{\circ} \overset{\circ}{\underset{\cdot}{\wedge}} = \bigwedge_{o < x \in H} x \overset{\circ}{\underset{\cdot}{\wedge}} \geq \overset{\circ}{\underset{\cdot}{\wedge}}$$

$$\text{box } H \xrightarrow[\text{isoton}]{\mathcal{V}} \mathbb{R}$$

$${}^H\mathcal{V} \text{ box} \Rightarrow \mathcal{V} \text{ stet}$$

$$a < o \Rightarrow {}^{<o}\mathcal{V} = {}^o\mathcal{V}$$

$$\not\Leftarrow {}^{<o}\mathcal{V} < {}^o\mathcal{V} \Rightarrow \begin{cases} H \ni x < o \\ o \leq x \in H \end{cases} \begin{matrix} x\mathcal{V} \leq {}^{<o}\mathcal{V} \\ x\mathcal{V} \geq {}^o\mathcal{V} \end{matrix} \Rightarrow \underbrace{{}^{<o}\mathcal{V} : {}^o\mathcal{V}} \cap {}^H\mathcal{V} = \emptyset \Rightarrow {}^H\mathcal{V} \text{ not box } \not\Leftarrow$$

$$o < b \Rightarrow {}^o\mathcal{V} = {}^{>o}\mathcal{V}$$

$$\not\Leftarrow {}^o\mathcal{V} < {}^{>o}\mathcal{V} \Rightarrow \begin{cases} H \ni x \leq o \\ o < x \in H \end{cases} \begin{matrix} x\mathcal{V} \leq {}^o\mathcal{V} \\ x\mathcal{V} \geq {}^{>o}\mathcal{V} \end{matrix} \Rightarrow \underbrace{{}^o\mathcal{V} : {}^{>o}\mathcal{V}} \cap {}^H\mathcal{V} = \emptyset \Rightarrow {}^H\mathcal{V} \text{ not box } \not\Leftarrow$$

$$\bigwedge_{\varepsilon} \begin{cases} > 0 \\ a < o: {}^o\mathcal{V} - \varepsilon < {}^o\mathcal{V} = {}^{<o}\mathcal{V} \\ o < b: {}^o\mathcal{V} + \varepsilon > {}^o\mathcal{V} = {}^{>o}\mathcal{V} \end{cases} \begin{matrix} \Rightarrow \text{not ob Schranke} \\ \Rightarrow \text{not unt Schranke} \end{matrix} \begin{matrix} \bigvee_{H \ni c < o} {}^o\mathcal{V} - \varepsilon < c\mathcal{V} \\ \bigvee_{o < d \in H} d\mathcal{V} < {}^o\mathcal{V} + \varepsilon \end{matrix}$$

$$\Rightarrow a < o < b: \bigwedge_{c \leq x \leq d} {}^o\mathcal{V} - \varepsilon < c\mathcal{V} \leq x\mathcal{V} \leq d\mathcal{V} < {}^o\mathcal{V} + \varepsilon \Rightarrow \overline{x\mathcal{V} - {}^o\mathcal{V}} < \varepsilon \Rightarrow \mathcal{V} \text{ stet in } o$$

$$a = o: \bigwedge_{a \leq x \leq d} a\mathcal{V} \leq x\mathcal{V} \leq d\mathcal{V} < a\mathcal{V} + \varepsilon \Rightarrow \overline{x\mathcal{V} - a\mathcal{V}} < \varepsilon \Rightarrow \mathcal{V} \text{ stet in } a$$

$$o = b: \bigwedge_{c \leq x \leq b} b\mathcal{V} - \varepsilon < c\mathcal{V} \leq x\mathcal{V} \leq \overline{x\mathcal{V} - b\mathcal{V}} < \varepsilon \Rightarrow \mathcal{V} \text{ stet in } b$$