

$$\frac{\begin{array}{c|c} \kappa\bar{\sigma} & -\kappa i\bar{\sigma} \\ \hline -j\bar{\sigma} & ij\bar{\sigma} \end{array}}{\quad} \xrightarrow[n_{\mathbb{H}_n^{\mathbb{C}}}{\times} \frac{\begin{array}{c|c} \kappa\sigma & \sigma j \\ \hline \kappa\sigma i & \sigma j i \end{array}}{\text{unit}} \xrightarrow{\begin{array}{c|c} 0 & j \\ \hline -j & 0 \end{array}} n_{\mathbb{H}_n^{\mathbb{D}}}$$

$$\begin{aligned} \Gamma \in n_{\mathbb{H}_n^{\mathbb{C}}} &\Rightarrow \frac{\begin{array}{c|c} \kappa\bar{\sigma} & -\kappa i\bar{\sigma} \\ \hline -j\bar{\sigma} & ij\bar{\sigma} \end{array}}{\quad} \downarrow_{\times} \frac{\begin{array}{c|c} \kappa\sigma & \sigma j \\ \hline \kappa\sigma i & \sigma j i \end{array}}{\text{unit}} \begin{array}{c|c} i & 0 \\ \hline 0 & i \end{array} \overbrace{\frac{\begin{array}{c|c} \kappa\bar{\sigma} & -\kappa i\bar{\sigma} \\ \hline -j\bar{\sigma} & ij\bar{\sigma} \end{array}}{\quad} \downarrow_{\times} \frac{\begin{array}{c|c} \kappa\sigma & \sigma j \\ \hline \kappa\sigma i & \sigma j i \end{array}}{\quad}}^* \\ &= \frac{\begin{array}{c|c} \kappa\bar{\sigma} & -\kappa i\bar{\sigma} \\ \hline -j\bar{\sigma} & ij\bar{\sigma} \end{array}}{\quad} \downarrow_{\times} \underbrace{\frac{\begin{array}{c|c} \kappa\sigma & \sigma j \\ \hline \kappa\sigma i & \sigma j i \end{array}}{\quad} \begin{array}{c|c} i & 0 \\ \hline 0 & i \end{array} \frac{\begin{array}{c|c} \kappa\bar{\sigma} & -\kappa i\bar{\sigma} \\ \hline -j\bar{\sigma} & ij\bar{\sigma} \end{array}}{\quad}}_{=\frac{\begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array}}{\quad}} \downarrow_{\times} \frac{\begin{array}{c|c} \kappa\sigma & \sigma j \\ \hline \kappa\sigma i & \sigma j i \end{array}}{\quad} = \frac{\begin{array}{c|c} \kappa\bar{\sigma} & -\kappa i\bar{\sigma} \\ \hline -j\bar{\sigma} & ij\bar{\sigma} \end{array}}{\quad} \downarrow_{\times} \underbrace{\frac{\begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array}}{\quad}}_{\downarrow_{\times}^{-1} \frac{\begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array}}{\quad}} \downarrow_{\times} \frac{\begin{array}{c|c} \kappa\sigma & \sigma j \\ \hline \kappa\sigma i & \sigma j i \end{array}}{\quad} \\ &= \frac{\begin{array}{c|c} \kappa\bar{\sigma} & -\kappa i\bar{\sigma} \\ \hline -j\bar{\sigma} & ij\bar{\sigma} \end{array}}{\quad} \underbrace{\downarrow_{\times}^{-1} \downarrow_{\times}^{-1}}_{=\frac{\begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array}}{\quad}} \frac{\begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array}}{\quad} \frac{\begin{array}{c|c} \kappa\sigma & \sigma j \\ \hline \kappa\sigma i & \sigma j i \end{array}}{\quad} = \frac{\begin{array}{c|c} \kappa\bar{\sigma} & -\kappa i\bar{\sigma} \\ \hline -j\bar{\sigma} & ij\bar{\sigma} \end{array}}{\quad} \frac{\begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array}}{\quad} \frac{\begin{array}{c|c} \kappa\sigma & \sigma j \\ \hline \kappa\sigma i & \sigma j i \end{array}}{\quad} = \frac{\begin{array}{c|c} i & 0 \\ \hline 0 & i \end{array}}{\quad} \\ \downarrow_{\times} \times \frac{\begin{array}{c|c} 1 & 0 \\ \hline 0 & -1 \end{array}}{\quad} = 0 &\Rightarrow \frac{\begin{array}{c|c} \kappa\bar{\sigma} & -\kappa i\bar{\sigma} \\ \hline -j\bar{\sigma} & ij\bar{\sigma} \end{array}}{\quad} \downarrow_{\times} \frac{\begin{array}{c|c} \kappa\sigma & \sigma j \\ \hline \kappa\sigma i & \sigma j i \end{array}}{\quad} \times \frac{\begin{array}{c|c} 0 & \kappa j \\ \hline -\kappa j & 0 \end{array}}{\quad} = 0 \\ &= \frac{\begin{array}{c|c} \kappa\bar{\sigma} & -\kappa i\bar{\sigma} \\ \hline -j\bar{\sigma} & ij\bar{\sigma} \end{array}}{\quad} \frac{\begin{array}{c|c} 1 & 0 \\ \hline 0 & -1 \end{array}}{\quad} \frac{\begin{array}{c|c} \kappa\sigma & \sigma j \\ \hline \kappa\sigma i & \sigma j i \end{array}}{\quad} \end{aligned}$$