$$ {}^{r}\mathbb{K}_{r}^{\mathtt{C}} $$

$$ \mathtt{C}|\mathtt{L} $$

$$ {}^{r}\mathbb{K}_{r}^{\mathtt{U}} \text{ unitary group} $$

$$ \mathtt{U}|\mathtt{L} $$

$$ \det \mathtt{L} = \sum_{\pi \in \mathcal{S}_n} -1^{\pi} \; {}_{1}\mathtt{L}^{\pi_1} \cdots {}_{n}\mathtt{L}^{\pi_n} $$

$$ {}^{I}\underline{\det}\ \mathtt{L} = \operatorname{tr} \mathtt{L} $$

$$ \mathtt{L}_t \in {}^{\mathtt{C}}_{n}\mathbb{K}^{n} $$

$$ \mathtt{L}_0 = I $$

$$ \partial_t^0 \mathtt{L}_t = \mathtt{L} $$

$$ \Rightarrow {}^{I}\underline{\det}\ \mathtt{L} = \partial_t^0 \det \mathtt{L}_t = \partial_t^0 \sum_{\pi \in \mathcal{S}_n} -1^{\pi} \; {}_{1}\mathtt{L}_t^{\pi_1} \cdots {}_{n}\mathtt{L}_t^{\pi_n} = \sum_{\pi \in \mathcal{S}_n} -1^{\pi} \partial_t^0 \; {}_{1}\mathtt{L}_t^{\pi_1} \cdots {}_{n}\mathtt{L}_t^{\pi_n} $$

$$ = \sum_{\pi \in \mathcal{S}_n} -1^{\pi} \underbrace{\partial_t^0 {}_{1}\mathtt{L}_t^{\pi_1} \cdots {}_{n}\mathtt{L}_t^{\pi_n} + \cdots + {}_{1}\mathtt{L}_t^{\pi_1} \cdots \partial_t^0 {}_{n}\mathtt{L}_t^{\pi_n}} $$

$$ = \sum_{\pi \in \mathcal{S}_n} -1^{\pi} \underbrace{{}_{1}\mathtt{L}^{\pi_1} \cdots {}_{n}\delta^{\pi_n} + \cdots + {}_{1}\delta^{\pi_1} \cdots {}_{n}\mathtt{L}^{\pi_n}} = {}_{1}\mathtt{L}^{1} + \cdots + {}_{n}\mathtt{L}^{n} = \operatorname{tr} \mathtt{L} $$

$$\frac{\flat \underbrace{\rceil}}{\underbrace{\rceil}} = \overline{\underbrace{\flat \rceil}\, \rceil^{-1}} : \quad \flat \underbrace{\rceil} = \underbrace{\rceil}\, \overline{\underbrace{\flat \rceil}\, \rceil^{-1}}$$

$$\flat \underbrace{\rceil} = \flat \sum_{\pi} \overset{\pi}{-1}\ {}_1\rceil^{\pi_1} \cdots {}_n\rceil^{\pi_n} = \sum_{\pi} \overset{\pi}{-1}\ \overbrace{\underbrace{\flat_1 \rceil^{\pi_1}} \cdots {}_n\rceil^{\pi_n}} + \cdots + \overbrace{{}_1\rceil^{\pi_1} \cdots \underbrace{\flat_n \rceil^{\pi_n}}}$$

$$= \sum_{\pi} \overset{\pi}{-1} \sum_{k} {}_1\rceil^{\pi_1} \cdots \underbrace{\flat_k \rceil^{\pi_k}} \cdots {}_n\rceil^{\pi_n}$$

$$= \sum_{k} \sum_{\ell} \underbrace{\flat_k \rceil^{\ell}} \sum_{\pi_k = \ell} \overset{\pi}{-1}\ {}_1\rceil^{\pi_1} \ \ddot{\wedge}\ {}_n\rceil^{\pi_n} = \sum_{k} \sum_{\ell} \underbrace{\flat_k \rceil^{\ell}}\ \overset{k+\ell}{-1} \sum_{\sigma} \overset{\sigma}{-1}\ {}_1\rceil^{\sigma_1} \ \ddot{\wedge}\ {}_n\rceil^{\sigma_n}$$

$$= \sum_{k} \sum_{\ell} \underbrace{\flat_k \rceil^{\ell}}\ \overset{k+\ell}{-1}\ \underbrace{{}_{N \llcorner k}\rceil^{N \llcorner \ell}} = \underbrace{\rceil} \sum_{k} \sum_{\ell} \underbrace{\flat_k \rceil^{\ell}}\ {}_{\ell}\rceil^{-1}{}_{k} = \underbrace{\rceil} \sum_{k} \underbrace{\flat_k \rceil}\ \rceil^{-1}{}_{k} = \underbrace{\rceil}\ \overline{\underbrace{\flat \rceil}\, \rceil^{-1}}$$

$$N \llcorner k \xrightarrow{\text{bij}} \sigma N \llcorner \ell$$

$$\overset{\pi}{-1} = \overset{\sigma}{-1}\ \overset{k+\ell}{-1}$$

$$\overset{\pi}{-1} = \prod_{i<j} \pi_i \sharp \pi_j = \prod_{k \neq i < j \neq k} \pi_i \sharp \pi_j \prod_{i<k} \pi_i \sharp \ell \prod_{k<j} \ell \sharp \pi_j$$

$$\prod_{k \neq i < j \neq k} \pi_i \sharp \pi_j = \overset{\sigma}{-1}$$

$$\prod_{i<k} \pi_i \sharp \ell \prod_{k<j} \ell \sharp \pi_j = \overset{k-1}{-1} \prod_{i<k} \ell \sharp \pi_i \prod_{k<j} \ell \sharp \pi_j = \overset{k-1}{-1} \prod_{m \neq k} \ell \sharp \pi_m = \overset{k-1}{-1}\ \overset{\ell-1}{-1} = \overset{k+\ell}{-1}$$