

$${}^n\mathbb{K}_n = NA {}^n\mathbb{K}_n^U$$

$$\Gamma = \begin{bmatrix} {}^1\Gamma \\ \cdot \\ {}^n\Gamma \end{bmatrix} \text{ free} \Rightarrow \bigvee_{\text{ONB}} \tilde{\Gamma} = \begin{bmatrix} {}^1\tilde{\Gamma} \\ \cdot \\ {}^n\tilde{\Gamma} \end{bmatrix}$$

$$\bigwedge_{m \leq n} \mathbb{K} \langle {}^1\Gamma \dots {}^m\Gamma \rangle = \mathbb{K} \langle {}^1\tilde{\Gamma} \dots {}^m\tilde{\Gamma} \rangle$$

$$\text{Ind } {}^j\tilde{\Gamma} = {}^j\Gamma - \sum_{i < j} {}^j\Gamma \star {}^i\tilde{\Gamma} \neq 0$$

$${}^j\tilde{\Gamma} = \underline{{}^j\Gamma} = {}^j\Gamma / \overline{{}^j\Gamma}$$

$$\Rightarrow {}^j\tilde{\Gamma} \star {}^j\tilde{\Gamma} = {}^j\Gamma \star {}^i\tilde{\Gamma} - {}^j\Gamma \star {}^i\tilde{\Gamma} = 0 = {}^j\tilde{\Gamma} \star {}^i\tilde{\Gamma} \Rightarrow {}^1\tilde{\Gamma} \dots {}^n\tilde{\Gamma} \text{ ONB}$$

$$\Rightarrow \bigvee \text{ ONB of } \Gamma$$

$${}^n\mathbb{K}_n \ni \Gamma = T\Lambda \tilde{\Gamma} \in NA {}^n\mathbb{K}_n^U$$

$$\begin{aligned} \Gamma \cdot \tilde{\Gamma}^* &= \begin{bmatrix} {}^1\Gamma \\ \cdot \\ {}^n\Gamma \end{bmatrix} \begin{bmatrix} {}^1\tilde{\Gamma}^* & \cdot & {}^n\tilde{\Gamma}^* \end{bmatrix} = \begin{bmatrix} {}^1\Gamma \star {}^1\tilde{\Gamma} & \cdot & {}^1\Gamma \star {}^n\tilde{\Gamma} \\ \cdot & \cdot & \cdot \\ {}^n\Gamma \star {}^1\tilde{\Gamma} & \cdot & {}^n\Gamma \star {}^n\tilde{\Gamma} \end{bmatrix} = \begin{bmatrix} {}^1\Gamma \star {}^1\tilde{\Gamma} & 0 & 0 & 0 \\ {}^2\Gamma \star {}^1\tilde{\Gamma} & {}^2\Gamma \star {}^2\tilde{\Gamma} & 0 & 0 \\ \cdot & \cdot & \cdot & 0 \\ {}^n\Gamma \star {}^1\tilde{\Gamma} & {}^n\Gamma \star {}^2\tilde{\Gamma} & \cdot & {}^n\Gamma \star {}^n\tilde{\Gamma} \end{bmatrix} \\ &= \begin{bmatrix} {}^1\Gamma_1 & 0 & 0 & 0 \\ {}^2\Gamma_1 & {}^2\Gamma_2 & 0 & 0 \\ \cdot & \cdot & \cdot & 0 \\ {}^n\Gamma_1 & {}^n\Gamma_2 & \cdot & {}^n\Gamma_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{{}^2\Gamma_1}{{}^1\Gamma_1} & 1 & 0 & 0 \\ \cdot & \cdot & \cdot & 0 \\ \frac{{}^n\Gamma_1}{{}^1\Gamma_1} & \frac{{}^n\Gamma_2}{{}^2\Gamma_2} & \frac{{}^n\Gamma_{n-}}{n-\Gamma_{n-}} & 1 \end{bmatrix} \begin{bmatrix} {}^1\Gamma_1 & 0 & 0 & 0 \\ 0 & {}^2\Gamma_2 & 0 & 0 \\ 0 & 0 & n-\Gamma_{n-} & 0 \\ 0 & 0 & 0 & {}^n\Gamma_n \end{bmatrix} \end{aligned}$$