$$X^{p^e} - 1 = \prod_{d \prec n} {}^{X}\hat{n} \text{ prime factors}$$

$${}^{X}\hat{n} = \sum_{p_1 \cdot\cdot p_k \prec n}^{\text{dist}} \overbrace{X^{n/p_1 \cdot\cdot p_k} - 1}^{-1^k} = \sum_{M \subset \text{ Trg } n} \overbrace{X^{n/\chi_M} - 1}^{-1^{|M|}} \in {}^{X}\blacktriangleright\mathbb{Z} \text{ irred}$$

$${}^{X}\hat{p} = \overbrace{X^{p/1} - 1}^{-1^0} \overbrace{X^{p/p} - 1}^{-1^1} = \frac{X^p - 1}{X - 1} = \sum_i^p X^i = {}^{X}_{\sharp}\chi^p$$

$${}^{X}\hat{p}^{1+k} = \overbrace{X^{p^{1+k}/1} - 1}^{-1^0} \overbrace{X^{p^{1+k}/p} - 1}^{-1^1} = \frac{X^{p^{1+k}} - 1}{X^{p^k} - 1} = \sum_i^p X^{p^k i}$$

$${}^{X}\hat{6} = \frac{\overbrace{X^6 - 1}\,\overbrace{X - 1}}{\underbrace{X^3 - 1}\,\underbrace{X^2 - 1}} = \frac{X^3 + 1}{X + 1} = X^2 - X + 1$$

$$m_i \geqslant 0 \Longrightarrow {}^{X}\overbrace{p_1^{1+m_1} \cdots p_k^{1+m_k}}^{\wedge} = {}^{X^{p_1^{m_1} \cdots p_k^{m_k}}}\overbrace{p_1 \cdots p_k}^{\wedge}$$