

$$\mathbb{1} = \mathbb{h}^{\leftarrow} \mathbb{1} \rightarrow \mathbb{K}$$

$$\mathbb{K} \triangleleft_{\mathbb{1}}^{\mathbb{h}} \mathbb{K} = \frac{\mathbb{K} \xleftarrow{\quad} \mathbb{1}}{\text{alg hom}} \quad \mathbb{1} \xrightarrow{\quad} \mathbb{K}$$

$$\dim_{\mathbb{h} \rightarrow \mathbb{K}} \mathbb{1} = \# \mathbb{h} \cap \overline{\mathbb{h}^{\leftarrow} \mathbb{1}} = \frac{\# \mathbb{h}}{\# \mathbb{h}^{\leftarrow} \mathbb{1}}$$

$$\mathbb{K} \triangleleft_{\mathbb{1}}^{\mathbb{h}} \mathbb{K} = \frac{\overline{\sigma}}{\sigma \in \mathbb{h}}$$

$$\Rightarrow (*) \quad \mathbb{h} \rightarrow \mathbb{K} \mid \mathbb{1} \subset \mathbb{K}$$

$$\mathbb{1} \xrightarrow{\quad} \mathbb{K} \triangleleft_{\mathbb{1}}^{\mathbb{h}} \mathbb{K} \leftarrow \mathbb{h} \times \mathbb{K} \triangleleft_{\mathbb{1}}^{\mathbb{h}} \mathbb{K} \ni \mathbb{1} \xrightarrow{\quad}$$

$$\mathbb{1} \overline{\mathbb{h}^{\leftarrow} \mathbb{1}} \in \mathbb{h} \cap \overline{\mathbb{h}^{\leftarrow} \mathbb{1}} \rightarrow \mathbb{K} \triangleleft_{\mathbb{1}}^{\mathbb{h}} \mathbb{K} \ni \mathbb{1} \xrightarrow{\quad}$$

$$\begin{array}{ccc} \mathbb{K} \triangleleft_{\mathbb{1}}^{\mathbb{h}} \mathbb{K} & \xleftarrow{\quad \times \quad} & \mathbb{h} \times \mathbb{K} \triangleleft_{\mathbb{1}}^{\mathbb{h}} \mathbb{K} \\ \uparrow \text{U} & & \uparrow \text{U} \\ \mathbb{h} \cap \overline{\mathbb{h}^{\leftarrow} \mathbb{1}} & \xleftarrow{\quad \times \quad} & \mathbb{h} \times \mathbb{h} \cap \overline{\mathbb{h}^{\leftarrow} \mathbb{1}} \end{array}$$

$$\Rightarrow \dim_{\mathbb{h} \rightarrow \mathbb{K}} \mathbb{1} \stackrel{\text{DED}}{\geq} \# \mathbb{K} \triangleleft_{\mathbb{1}}^{\mathbb{h}} \mathbb{K} \geq \# \mathbb{h} \cap \overline{\mathbb{h}^{\leftarrow} \mathbb{1}} = \frac{\# \mathbb{h}}{\# \mathbb{h}^{\leftarrow} \mathbb{1}} = \dim_{\mathbb{h} \rightarrow \mathbb{K}} \mathbb{h}^{\leftarrow} \mathbb{1} \rightarrow \mathbb{K} \geq_* \dim_{\mathbb{h} \rightarrow \mathbb{K}} \mathbb{1}$$

$$\Rightarrow \dim_{\mathbb{h} \rightarrow \mathbb{K}} \mathbb{1} = \dim_{\mathbb{h} \rightarrow \mathbb{K}} \mathbb{h}^{\leftarrow} \mathbb{1} \rightarrow \mathbb{K} \Rightarrow \mathbb{1} = \mathbb{h}^{\leftarrow} \mathbb{1} \rightarrow \mathbb{K}$$

$$\# \mathbb{K} \triangleleft_{\mathbb{1}}^{\mathbb{h}} \mathbb{K} = \# \mathbb{h} \cap \overline{\mathbb{h}^{\leftarrow} \mathbb{1}} = \frac{\# \mathbb{h}}{\# \mathbb{h}^{\leftarrow} \mathbb{1}} \Rightarrow \mathbb{h} \cap \overline{\mathbb{h}^{\leftarrow} \mathbb{1}} \xrightarrow{(\circ)} \mathbb{K} \triangleleft_{\mathbb{1}}^{\mathbb{h}} \mathbb{K}$$

