

$$\begin{aligned}
& \mathbb{1} \in \mathbb{N}\mathbb{K} \\
& \mathbb{1} \xleftarrow{\sigma} \mathbb{1} \\
& \text{lin autom} \\
\mathbf{C}(\mathbb{1}) &= \frac{\sigma \mathbb{1} + \mathbb{1}}{\sigma \mathbb{1} + \sigma \mathbb{1}: \sigma x \cdot \mathbb{1} = \sigma x \cdot \sigma \mathbb{1}} \\
\mathbb{T}_{\mathbb{H}} \mathbb{1} &= \frac{\mathbb{1} \in \mathbb{1}}{\bigwedge_{\sigma} \sigma \mathbb{1} = \mathbb{1}} \subset \mathbb{1} \text{ fix lin}
\end{aligned}$$

$$\frac{\mathbb{T}_{\mathbb{H}} \mathbb{1}}{\mathbb{T}_{\mathbb{H}} \mathbb{K}} = \frac{\mathbb{1}}{\mathbb{K}}: \dim_{\mathbb{T}_{\mathbb{H}} \mathbb{K}} \mathbb{T}_{\mathbb{H}} \mathbb{1} = \dim_{\mathbb{K}} \mathbb{1}$$

$$\begin{aligned}
\mathbb{T}_{\mathbb{H}} \mathbb{1} &= \mathcal{B}_{\text{free}} \cdot \mathbb{T}_{\mathbb{H}} \mathbb{K} \Rightarrow \mathbb{1} = \mathcal{B}_{\text{free}} \cdot \mathbb{K} \\
\text{free}_{\mathbb{K}} \mathbb{1}^0 \dots \mathbb{1}^n \in \mathbb{T}_{\mathbb{H}} \mathbb{1} &\stackrel{(!)}{\Rightarrow} \mathbb{1}^0 \dots \mathbb{1}^n \text{ free}_{\mathbb{K}}
\end{aligned}$$

Ind<sub>n ≥ 0</sub>:

$$n = 0: \mathbb{1}^0 \text{ free}_{\mathbb{K}} \Rightarrow \mathbb{1}^0 \neq 0 \Rightarrow \mathbb{1}^0 \text{ free}_{\mathbb{K}}$$

$$0 \leq n \rightsquigarrow n+1: \text{free}_{\mathbb{K}} \mathbb{1}^0 \dots \mathbb{1}^n \in \mathbb{T}_{\mathbb{H}} \mathbb{1}$$

$$\neg \text{ not } \mathbb{1}^0 \dots \mathbb{1}^n \text{ free}_{\mathbb{K}} \Rightarrow 0 = \sum_{0 \leq i \leq n} \mathbb{1}^i \cdot \mathbb{1} = \mathbb{1}^j \underbrace{\mathbb{1}}_{\neq 0} + \sum_{i \neq j} \mathbb{1}^i \cdot \mathbb{1} \Rightarrow 0 = \mathbb{1}^j + \sum_{i \neq j} \mathbb{1}^i \cdot \frac{\mathbb{1}}{\mathbb{1}}$$

$$\bigwedge_{\sigma} 0 = \sigma \left( \mathbb{1}^j + \sum_{i \neq j} \mathbb{1}^i \cdot \frac{\mathbb{1}}{\mathbb{1}} \right) = \sigma \mathbb{1}^j + \sum_{i \neq j} \sigma \mathbb{1}^i \cdot \frac{\sigma \mathbb{1}}{\sigma \mathbb{1}} = \mathbb{1}^j + \sum_{i \neq j} \mathbb{1}^i \cdot \frac{\sigma \mathbb{1}}{\sigma \mathbb{1}}$$

$$\Rightarrow 0 = \sum_{i \neq j} \mathbb{1}^i \cdot \underbrace{\frac{\mathbb{1}}{\mathbb{1}} - \frac{\sigma \mathbb{1}}{\sigma \mathbb{1}}}_{\text{ind}} \Rightarrow \bigwedge_{i \neq 0, j} \frac{\mathbb{1}}{\mathbb{1}} = \frac{\sigma \mathbb{1}}{\sigma \mathbb{1}} \Rightarrow \frac{\mathbb{1}}{\mathbb{1}} \in \mathbb{T}_{\mathbb{H}} \mathbb{K} = \mathbb{K} \ni 1$$

$$0 = \mathbb{1}^j + \sum_{i \neq j} \mathbb{1}^i \cdot \frac{\mathbb{1}}{\mathbb{1}} \Rightarrow \text{not free}_{\mathbb{K}} \mathbb{1}^0 \dots \mathbb{1}^n \quad \spadesuit$$