

$$\mathbb{O} = \mathbb{H} + \ell\mathbb{H}$$

$$x + \ell y \in \mathbb{O} \rightarrow {}_2\mathbb{H}^2 \ni \frac{x \mid y}{-y^* \mid x^*}$$

$$\underbrace{x + \ell y} \underbrace{u + \ell v} = \frac{x \mid y}{-y^* \mid x^*} \frac{u \mid v}{-v^* \mid u^*} = \underbrace{xu - yv^*} + \ell \underbrace{xv + yu^*}$$