

$${}_z^w \vartheta = \sum_{\nu} \pi i \nu w \mathbf{e}^{2\pi i z \nu} \mathbf{e}^{\pi i \nu^2 w}$$

$$w = u + iv \Rightarrow iw = iu - v$$

$$z = x + iy \Rightarrow iz = ix - y$$

$$\overline{\pi i n^2 w \mathbf{e}^{2\pi i n z}} \mathbf{e} = -\pi n^2 v \mathbf{e}^{-2\pi n y} \mathbf{e}^{i\pi n^2 x}$$

$${}_{z+\kappa w+k}^w \vartheta = \mathbf{e}^{-\pi i \kappa w \kappa - 2\pi i z \kappa} \mathbf{e}^{\pi i \kappa^2 w} {}_z^w \vartheta$$