

$$\mathbb{C} \supset \mathfrak{h} \text{ prim}_0 \Rightarrow \Leftrightarrow$$

$$(1) \mathfrak{h} \text{ prim}_1$$

$$(2) \bigwedge \mathfrak{L} \in \mathbb{S}_{\Delta_0}^{\mathfrak{h}} \bigwedge_{\circ}^{\mathbb{C} \perp \mathfrak{h}} \text{deg}_{\mathfrak{L}} = 0$$

$$(3) \bar{\mathbb{C}} \perp \mathfrak{h} \text{ prim}_0$$

$$(4) \mathfrak{h} \triangleleft_{\omega} \mathbb{C} \xleftarrow[\text{hull}]{e} \mathbb{C} \blacktriangleright \mathbb{C}$$

$$(5) \bigwedge \gamma \in \mathfrak{h} \triangleleft_{\omega} \mathbb{C} \bigwedge \mathfrak{L} \in \mathbb{S}_{\Delta_0}^{\mathfrak{h}} \int_{dw/2\pi i}^{\mathfrak{L}} \gamma = 0$$

$$(6) \mathfrak{h} \triangleleft_{\omega} \mathbb{C} \xleftarrow[\text{surj}]{\frac{d}{dz}} \mathfrak{h} \triangleleft_{\omega} \mathbb{C}$$

$$(7) \begin{array}{ccc} & \vee \tilde{\gamma} & \rightarrow \mathbb{C} \\ \mathfrak{h} & \nearrow & \downarrow \text{exp} \\ & \wedge \gamma & \rightarrow \mathbb{C}^{\times} \end{array}$$

$$(8) \begin{array}{ccc} & \vee \tilde{\gamma} & \rightarrow \mathbb{C}^{\times} \\ \mathfrak{h} & \nearrow & \downarrow ()^2 \\ & \wedge \gamma & \rightarrow \mathbb{C}^{\times} \end{array}$$

$$(9) \mathfrak{h} \underset{\text{holeo}}{\simeq} \mathbb{C}$$

$$(1) \rightarrow (2): \mathfrak{L} \in \mathbb{S}_{\triangleleft} \mathfrak{H} \Rightarrow \mathfrak{L} \text{ null-htp } \mathfrak{H} \Rightarrow \mathfrak{L} \text{ null-hlg } \mathfrak{H} \Rightarrow \text{int } \mathbb{C} \setminus \mathfrak{H} \bigwedge_o^{\mathbb{C} \setminus \mathfrak{H}} \text{deg}_{\mathfrak{L}} = 0$$

$$(2) \rightarrow (3): \mathfrak{L} \bar{\mathbb{C}} \setminus \mathfrak{H} = \underset{\neq \emptyset}{A} \dot{\cup} B$$

$$B \subset \bar{\mathbb{C}} \setminus \mathfrak{H} \subset \bar{\mathbb{C}} \Rightarrow T \text{ cpt } B$$

$$\infty \in B \Rightarrow \text{cpt } A \subset \mathfrak{H} \cup A = \bar{\mathbb{C}} \setminus B \subset \mathbb{C}$$

$$\underbrace{\mathfrak{H} \cup A \setminus A = \mathfrak{H}}_{\text{LEM}} \Rightarrow \bigvee_{\text{closed polygons}} \mathfrak{L}_j \in \mathbb{R}_{\triangleleft} \mathfrak{H}$$

$$\bigwedge_z^A 1 = \sum_j \int_{dw/2\pi i}^{\mathfrak{L}_j} \frac{w}{w-z} = \sum_j {}^z \text{deg}_{\mathfrak{L}_j} \Rightarrow \bigvee_j {}^z \text{deg}_{\mathfrak{L}_j} \neq 0 \quad \ddagger$$

$$(3) \rightarrow (4): \mathbb{C} \setminus \mathfrak{H} \ni \infty \text{ comp-hull} \xrightarrow{\text{oRUN}} \mathfrak{H}_{\triangleleft} \mathbb{C} \xleftarrow[\text{hull}]{\varrho} \bar{\mathbb{C}}_{\triangleleft} \bar{\mathbb{C}} = \mathbb{C}_{\triangleleft} \mathbb{C}$$

$$(4) \rightarrow (5): \bigwedge 1 \in \mathbb{C}_{\triangleleft} \mathbb{C} \int_{dw/2\pi i}^{\mathfrak{L}} 1 = 0 \Rightarrow 0 = \int_{dw/2\pi i}^{\mathfrak{L}} 1_n \rightsquigarrow \int_{dw/2\pi i}^{\mathfrak{L}} \gamma = 0$$

$$(5) \rightarrow (6): o \in \mathfrak{H}$$

$$\gamma \in \mathfrak{H}_{\triangleleft} \mathbb{C} \xrightarrow{5} \text{well-def } {}^z F = \int_{dw/2\pi i}^{o|z} \gamma \Rightarrow F \in \mathfrak{H}_{\triangleleft} \mathbb{C}$$

$$\underline{F} = \gamma$$

$$(6) \rightarrow (7): \gamma \in \mathfrak{H}_{\triangleleft} \mathbb{C}^{\times} \Rightarrow \frac{\gamma}{\gamma} \in \mathfrak{H}_{\triangleleft} \mathbb{C} \Rightarrow \bigvee F \in \mathfrak{H}_{\triangleleft} \mathbb{C}$$

$$\underline{F} = \frac{\gamma}{\gamma} \Rightarrow \frac{d \exp F}{dz} \frac{1}{\gamma} = \gamma^{-2} \left(\exp F \frac{\gamma}{\gamma} - \exp F \underline{\gamma} \right) = 0 \Rightarrow \frac{\exp F}{\gamma} = \text{cst} = e^a \Rightarrow \gamma = \exp(F - a)$$

$$(7) \rightarrow (8): \exp F = \gamma \Rightarrow (\exp F/2)^2 = \exp F = \gamma$$

$$(8) \rightarrow (9) \text{ Riemann mapping theorem } \mathbb{C} \supset \mathfrak{H}_{01} \text{ prim} \Rightarrow \mathfrak{H} \asymp \mathbb{C}$$

$$(9) \rightarrow (1): \mathbb{C}_{1} \text{ prim} \Rightarrow \mathfrak{H}_{1} \text{ prim}$$