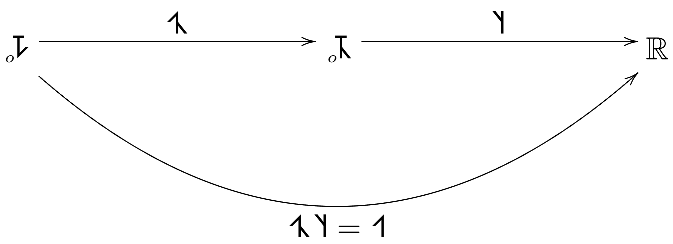

$$\square = \sum_{m_1+2m_2+3m_3\cdots=n} \begin{bmatrix} m_1+m_2+\cdots \\ m_1, m_2, \cdots \end{bmatrix} \square^{m_1} \square^{m_2} \cdots \frac{\square_{m_1+m_2+\cdots}}{m_1+m_2+\cdots!}$$

$$\mathsf{L}^n \square = \sum_{m_1+2m_2+3m_3\cdots=n} \begin{bmatrix} m_1+m_2+\cdots \\ m_1, m_2, \cdots \end{bmatrix} \mathsf{L}^1 \square^{m_1} \mathsf{L}^2 \square^{m_2} \cdots \frac{\square_{m_1+m_2+\cdots}}{m_1+m_2+\cdots!}$$

$$= \sum_{m_1+2m_2+3m_3\cdots=n} \frac{1}{1!}\overbrace{\mathsf{L}^{1o}\square_1}^{m_1} \frac{1}{2!}\overbrace{\mathsf{L}^{2o}\square_2}^{m_2} \cdots \frac{\square_{m_1+m_2+\cdots}}{m_1!m_2!\cdots}$$

$$\underbrace{\mathsf{L}^1\cdots\mathsf{L}^k}\, {}^0\mathfrak{g}_w \ltimes \square_k = \sum_{1\leqslant n\leqslant k} \sum_{|\beta|=k}^{\mathbb{N}^n_>} \overbrace{\underbrace{\mathsf{L}^{\sigma_1}\cdots\mathsf{L}^{\sigma_{\beta_1}}}\, {}^0\mathfrak{g}_w}_{\beta_1} \boxtimes\cdots\boxtimes \overbrace{\underbrace{\mathsf{L}^{\sigma_{k+1-\beta_n}}\cdots\mathsf{L}^{\sigma_k}}\, {}^0\mathfrak{g}_w}_{\beta_n} {}^w\square_n$$

$$\mathsf{L}^k\, {}^0\mathfrak{g}_w \ltimes \square_k = k! \sum_{1\leqslant n\leqslant k} \sum_{|\beta|=k}^{\mathbb{N}^n_>} \overbrace{\mathsf{L}^{\beta_1}\, {}^0\mathfrak{g}_w}_{\beta_1} \boxtimes\cdots\boxtimes \overbrace{\mathsf{L}^{\beta_n}\, {}^0\mathfrak{g}_w}_{\beta_n} {}^w\square_n$$

$$\mathsf{L}^n \square_n = \sum_{0\leqslant m\leqslant n} \begin{bmatrix} n \\ m \end{bmatrix} \overbrace{\mathsf{L}^m \square_m} \overbrace{\mathsf{L}^{n-m} \square_{n-m}}$$