

$${}_z\check{C}^\lambda = \frac{\lambda i + 1/2}{1 - \bar{z}z} \frac{\lambda i + 1/2}{\frac{z\bar{z}}{1}} = \frac{\lambda i + 1/2 - \lambda i + 1/2}{\frac{z\bar{z}}{z\bar{z} - 1}}$$

$$\begin{aligned} \text{LHS} &= \int_{du}^{\mathbb{T}} \left(\frac{1 - \bar{z}^2}{\bar{u} - z} \right)^{\lambda i + 1/2} = \int_{dv/2\pi}^{0|2\pi} \left(\frac{1 - \bar{z}^2}{iv \mathbf{e} - z} \right)^{\lambda i + 1/2} = (1 - \bar{z}^2)^{\lambda i + 1/2} \int_{dv/2\pi}^{0|2\pi} \overline{1 - -iv \mathbf{e}z}^{-2\lambda i - 1} \\ &= (1 - \bar{z}^2)^{\lambda i + 1/2} \int_{dv/2\pi}^{0|2\pi} \sum_{n \geq 0} \frac{(\lambda i + 1/2)_{n - niv} \mathbf{e}z^n}{n!} \stackrel{\text{orth}}{=} (1 - \bar{z}^2)^{\lambda i + 1/2} \sum_{n \geq 0} \frac{(\lambda i + 1/2)_n^2}{(n!)^2} \bar{z}^{2n} = \text{RHS} \end{aligned}$$

$${}_z\check{C}^{\lambda i + 1/2} = (1 - \bar{z}^2)^{\lambda i + 1/2} z \bar{z} \left[\lambda i + 1/2 : \lambda i + 1/2 \right]_1 = \sum_{j+k=n} \frac{(-1/2 - \lambda i)_j}{j!} \frac{(1/2 + \lambda i)_k^2}{(k!)^2}$$

$$\text{Harish-Chandra } \check{C}_\lambda^{-2} = \pi \lambda \tanh(\pi \lambda)$$

$$\frac{\Gamma_{\lambda i + 1/2} \Gamma_{-\lambda i + 1/2}}{\Gamma_{2\lambda i}} = 4\pi \lambda \tanh(\pi \lambda)$$

$${}_z\varphi^\lambda = \int_{dw}^S \frac{1 - \frac{2}{z}}{z - w} = \frac{1 - \frac{2}{z}}{\bar{z}^2} \left[\begin{matrix} 1 + i\lambda : 1 + i\lambda \\ 1 \end{matrix} \right] = \frac{1 - \frac{2}{z}}{-\frac{2}{z} / 1 - \frac{2}{z}} \left[\begin{matrix} 1 + i\lambda : 1 - i\lambda \\ 1 \end{matrix} \right]$$