

$$\beta_1 - \beta_0 \notin \mathbb{N}$$

$$2x^2 \underline{1} + \underline{1-x} \underline{1} - 2\underline{1} = 0 \begin{cases} \gamma_0 = 1 + 2x + \frac{1}{3}x^2 & \beta_0 = 0 \\ \gamma_1 = 3x^{1/2} \sum_n^{\mathbb{N}} \frac{(-x/2)^{\underline{n}}}{(2n-3)(2n-1)(2n+1)} & \beta_1 = 1/2 \end{cases}$$

$$4x^2 \underline{1} + 4x \underline{1} + \underline{4x^2-1} \underline{1} = 0 \begin{cases} \gamma_0 = \sin x^{-1/2} & \beta_0 = \\ \gamma_1 = \cos x^{-1/2} & \beta_1 = \end{cases}$$

$$4x^2 \underline{1} + 4x \underline{1} - \underline{4x^2+1} \underline{1} = 0 \begin{cases} \gamma_0 = \sinh x^{-1/2} & \beta_0 = \\ \gamma_1 = \cosh x^{-1/2} & \beta_1 = \end{cases}$$

$$2x \underline{x+1} \underline{1} + 3 \underline{x+1} \underline{1} - \underline{1} = 0 \begin{cases} \gamma_0 = \sum_n^{\mathbb{N}} \frac{(-x)^n}{(1-2n)(1+2n)} & \beta_0 = \\ \gamma_1 = x^{-1/2} + x^{1/2} & \beta_1 = \end{cases}$$

$$4x \underline{1} + 3 \underline{1} + 3 \underline{1} = 0 \begin{cases} \gamma_0 = x^{1/4} \sum_n^{\mathbb{N}} \frac{(-3x/4)^{\underline{n}}}{(5/4)_n} & \beta_0 = \\ \gamma_1 = \sum_n^{\mathbb{N}} \frac{(-3x/4)^{\underline{n}}}{(3/4)_n} & \beta_1 = \end{cases}$$

$$2x^2 \underline{1-x} \underline{1} - x \underline{1+7x} \underline{1} + \underline{1} = 0 \begin{cases} \gamma_0 = \frac{1}{15} \sum_n^{\mathbb{N}} x^{n+1} (2n+3)(2n+5) & \beta_0 = \\ \gamma_1 = \frac{1}{2} \sum_n^{\mathbb{N}} x^{n+1/2} (n+1)(n+2) & \beta_1 = \end{cases}$$

$$2x \underline{1} + 5 \underline{1-2x} \underline{1} - 5 \underline{1} = 0 \begin{cases} \gamma_0 = 3 \sum_n^{\mathbb{N}} \frac{(5x)^{\underline{n}}}{(2n+1)(2n+3)} & \beta_0 = \\ \gamma_1 = x^{-3/2} + 10x^{-1/2} & \beta_1 = \end{cases}$$

$$8x^2 \underline{1} + 10x \underline{1} + \underline{1+x} \underline{1} = 0 \begin{cases} \gamma_0 = x^{1/4} \sum_n^{\mathbb{N}} \frac{(x/8)^{\underline{n}}}{(7/4)_n} & \beta_0 = \\ \gamma_1 = x^{-1/2} \sum_n^{\mathbb{N}} \frac{(x/8)^{\underline{n}}}{(1/4)_n} & \beta_1 = \end{cases}$$

$$3x \underline{1} + \underline{2-x} \underline{1} - 2 \underline{1} = 0 \begin{cases} \gamma_0 = x^{1/3} \sum_n^{\mathbb{N}} (x/3)^{\underline{n}} (1+3n/4) & \beta_0 = \\ \gamma_1 = \sum_n^{\mathbb{N}} (x/3)^n \frac{n+1}{(2/3)_n} & \beta_1 = \end{cases}$$

$$2x \underline{x+3} \underline{\quad} - 3 \underline{x+1} \underline{\quad} + 2\gamma = 0 \begin{cases} \gamma_0 = 3 \sum_n^{\mathbb{N}} x^{n+3/2} \frac{(-1/3)^n}{(1-2n)(2n+1)(2n+3)} & \beta_0 = \\ \gamma_1 = 1 + \frac{2}{3}x + \frac{1}{9}x^2 & \beta_1 = \end{cases}$$

$$2x \underline{\quad} + \underline{1-2x^2} \underline{\quad} - 4x\gamma = 0 \begin{cases} \gamma_0 = x^{1/2} \exp x^2/2 & \beta_0 = \\ \gamma_1 = \sum_n^{\mathbb{N}} \frac{(x^2/2)^n}{(3/4)_n} & \beta_1 = \end{cases}$$

$$x \underline{4-x} \underline{\quad} + \underline{2-x} \underline{\quad} + 4\gamma = 0 \begin{cases} \gamma_0 = \frac{1}{3} x^{1/2} \sum_n^{\mathbb{N}} (x/4)^{\times} (2n+3) (-3/2)_n & \beta_0 = \\ \gamma_1 = 1 - 2x + x^2/2 & \beta_1 = \end{cases}$$

$$3x^2 \underline{\quad} + x \underline{\quad} - \underline{1+x} \gamma = 0 \begin{cases} \gamma_0 = x \sum_n^{\mathbb{N}} (x/3)^{\times} \frac{1}{(7/3)_n} & \beta_0 = \\ \gamma_1 = x^{-1/3} \sum_n^{\mathbb{N}} (x/3)^{\times} \frac{1}{(-1/3)_n} & \beta_1 = \end{cases}$$

$$2x \underline{\quad} + \underline{1+2x} \underline{\quad} + 4\gamma = 0 \begin{cases} \gamma_0 = x^{1/2} \sum_n^{\mathbb{N}} (-x)^{\times} (1+2n/3) & \beta_0 = \\ \gamma_1 = \sum_n^{\mathbb{N}} x^n \frac{{}_n n_1 (n+1)}{1/2_n} & \beta_1 = \end{cases}$$

$$2x \underline{\quad} + \underline{1+2x} \underline{\quad} - 5\gamma = 0 \begin{cases} \gamma_0 = x^{1/2} + \frac{4}{3}x^{3/2} + \frac{4}{15}x^{5/2} & \beta_0 = \\ \gamma_1 = 15 \sum_n^{\mathbb{N}} \frac{(-x)^{\times}}{(1-2n)(3-2n)(5-2n)} & \beta_1 = \end{cases}$$

$$2x^2 \underline{\quad} - 3x \underline{1-x} \underline{\quad} + 2\gamma = 0 \begin{cases} \gamma_0 = x^2 \sum_n^{\mathbb{N}} (-3x/2)^n \frac{n+1}{(5/2)_n} & \beta_0 = \\ \gamma_1 = x^{1/2} \sum_n^{\mathbb{N}} (-3x/2)^{\times} (1-2n) & \beta_1 = \end{cases}$$

$$2x^2 \underline{\quad} + x \underline{4x-1} \underline{\quad} + 2 \underline{3x-1} \gamma = 0 \begin{cases} \gamma_0 = x^2 e^{-2x} & \beta_0 = \\ \gamma_1 = x^{-1/2} \sum_n^{\mathbb{N}} \frac{(-2x)^n}{(-3/2)_n} & \beta_1 = \end{cases}$$

$$2x \underline{\quad} - \underline{1+2x^2} \underline{\quad} - x\gamma = 0 \begin{cases} \gamma_0 = \sum_m^{\mathbb{N}} x^{2m+3/2} \frac{1}{2^m (7/4)_m} & \beta_0 = \\ \gamma_1 = \exp x^2/2 & \beta_1 = \end{cases}$$