

$$\alpha \in \mathbb{R}$$

$$\mathbb{Q}_\alpha^D = \frac{p/q \in \mathbb{Q}}{\alpha - p/q < 1/q^2}$$

$$\mathbb{Q}_\alpha^H = \frac{p/q \in \mathbb{Q}}{\sqrt{5}\alpha - p/q < 1/q^2}$$

$$\alpha \in \mathbb{Q} \Leftrightarrow \mathbb{Q}_\alpha^D \text{ finite} \Leftrightarrow \mathbb{Q}_\alpha^H \text{ finite}$$