$$[a_0\ \ a_1\ \cdots\ a_n] = a_0 + \frac{1}{[a_1 \cdots a_n]} = a_0 + \left(a_1 + \left(a_2 + \cdots\left(a_{n-1} + a_n^{-1}\right)^{-1}\cdots\right)^{-1}\right)^{-1}$$

$$\begin{array}{c|c} a_0 & 1 \\ \hline 1 & 0 \end{array}\ \begin{array}{c|c} a_1 & 1 \\ \hline 1 & 0 \end{array} \cdots \begin{array}{c|c} a_n & 1 \\ \hline 1 & 0 \end{array} = \begin{array}{c|c} p_n & p_{n-1} \\ \hline q_n & q_{n-1} \end{array} \Longrightarrow \begin{cases} \dfrac{p_n}{q_n} = [a_0\ a_1\ \cdots\ a_n] \\ \dfrac{p_n}{p_{n-1}} = [a_n\ \cdots\ a_1\ a_0] \\ \dfrac{q_n}{q_{n-1}} = [a_n\ \cdots\ a_1] \end{cases}$$

$$\begin{array}{c|c} a_0 & 1 \\ \hline 1 & 0 \end{array}\ \begin{array}{c|c} a_1 & 1 \\ \hline 1 & 0 \end{array} \cdots \begin{array}{c|c} a_{n+1} & 1 \\ \hline 1 & 0 \end{array} = \begin{array}{c|c} p_n & p_{n-1} \\ \hline q_n & q_{n-1} \end{array} \begin{array}{c|c} a_{n+1} & 1 \\ \hline 1 & 0 \end{array} = \begin{array}{c|c} p_n a_{n+1} + p_{n-1} & p_n \\ \hline q_n a_{n+1} + q_{n-1} & q_n \end{array}$$

$$\Longrightarrow \begin{cases} p_{n+1} = p_n a_{n+1} + p_{n-1} \\ q_{n+1} = q_n a_{n+1} + q_{n-1} \end{cases}$$

$$\frac{p_{n+1}}{q_{n+1}} = \frac{p_n a_{n+1} + p_{n-1}}{q_n a_{n+1} + q_{n-1}} = \frac{\overbrace{p_{n-1} a_n + p_{n-2}}\, a_{n+1} + p_{n-1}}{\underbrace{q_{n-1} a_n + q_{n-2}}\, a_{n+1} + q_{n-1}} = \frac{p_{n-1}\overbrace{a_n a_{n+1} + 1} + p_{n-2} a_{n+1}}{q_{n-1}\underbrace{a_n a_{n+1} + 1} + q_{n-2} a_{n+1}}$$

$$= \frac{p_{n-1}\left(a_n + a_{n+1}^{-1}\right) + p_{n-2}}{q_{n-1}\left(a_n + a_{n+1}^{-1}\right) + q_{n-2}} \underset{\text{Ind}}{=} [a_0\ \cdots a_{n-2} | a_n + a_{n+1}^{-1}] = [a_0\ \ a_1\ \cdots\ a_{n+1}]$$

$$\mathbb{Q} = \left\{[a_0\ a_1\ \cdots\ a_m]\right\}$$

$$\overline{\begin{array}{c|c} p_n & p_{n-1} \\ \hline q_n & q_{n-1} \end{array}} = p_n\, q_{n-1} - p_{n-1}\, q_n = (-1)^{n-1}$$

$$[a_0\ \ a_1\ \cdots\ a_m\overline{b_0\ b_1\ \cdots\ b_n}]\ \text{quadratic irrat}$$

$$\text{1-per}\ \begin{cases}\sqrt{2}=[1\ \bar{2}]\\ \dfrac{\sqrt{5}+1}{2}=[\bar{1}]\quad \dfrac{\sqrt{5}-1}{2}=[0\ \bar{1}]\end{cases}$$

$$\text{1-per}\ \sqrt{k^2+1}=[k\ \overline{2k}]$$

$$\text{2-per}\ \sqrt{k^2+2}=[k\ \overline{k\ 2k}]$$

$$\pi=[3\ 7\ 15\ 1\ 292\ \cdots]$$

$$\frac{{}^{2/k}\mathfrak{e}+1}{{}^{2/k}\mathfrak{e}-1}=[(2n+1)\,k]$$

$$\left.\begin{array}{c|c}1&1\\\hline 1&0\end{array}\right.^{n}=\begin{array}{c|c}F_{n+1}&F_n\\\hline F_n&F_{n-1}\end{array}$$

$$\sqrt{d}=[a_0\ \overline{\underbrace{a_1\ \cdots\ a_m\ \cdots a_1}_{\text{palindrom}}\ 2a_0}]$$

$$[a_0\ \ a_1\ \cdots\ a_n\ \cdots]-[a_0\ \ a_1\ \cdots\ a_n]=\frac{(-1)^n}{q_n\left(q_n a_{n+1}+q_{n-1}\right)}$$

$$\left|[a_0\ a_1\ \cdots\ a_n\ \cdots]-[a_0\ a_1\ \cdots\ a_n]\right|\underset{\text{DIR}}{<}\frac{1}{q_n^2 a_{n+1}}\leqslant\frac{1}{2q_n^2}$$