

$$\text{Riemann } \zeta_s = \sum_{n \geq 1} \frac{1}{n^s} = \prod_p \frac{p^s}{p^s - 1} \in \mathbb{C}^{\times} \setminus \{1\}$$

$$\xi_s = \frac{\Gamma(s/2)}{\pi^{s/2}} \zeta_s \in \mathbb{C}^{\times}$$

$$\xi_s = \xi_{1-s}$$

$$\overline{\sum_{p \leq x} \frac{1}{p}} \ll \text{const}$$

$$\text{Dirichlet } \zeta_s^\chi = \sum_{n \geq 1} \frac{n^\chi}{n^s} = \prod_p \frac{p^s}{p^s - p^\chi} \in \mathbb{C}^{\times}$$

$$\xi_s^\chi = q^{s/2} \frac{\Gamma(s/2 + (1-\chi)/4)}{\pi^{s/2 + (1-\chi)/4}} \zeta_s^\chi$$

$$\xi_s^\chi \sqrt{-1}^\chi q = \bar{\xi}_{1-s} \sum_{k \in \mathbb{Z} \setminus \mathbb{Z}q} e^{2\pi i k/q} \chi_k$$

$$a \wedge q = 1 \Rightarrow \overline{\sum_{p \leq x} \frac{1}{p} - \frac{1}{\mathbb{Z} \setminus \mathbb{Z}q}} \ll \text{const}$$

Dedekind $Q \subset \mathbb{Q}$

$$\#Q_s = \sum_{n \geq 1} \frac{n^Q}{n^s} = \sum_{0 \neq n \in \mathbb{Z}} \frac{1}{n^s} = \prod_p \frac{p^s}{p^s - 1} \in \mathbb{C}^{\times} \setminus \{1\} \subset \mathbb{C}$$

$$\circlearrowleft Q_s = \overline{\det \omega_i^s} \left(\frac{\Gamma_{s/2}}{\pi^{s/2}} \right)^{r_{\mathbb{R}}} \left(\frac{\Gamma_s}{(2\pi)^s} \right)^{r_{\mathbb{C}}} \#Q_s$$

$$\circlearrowleft Q_s = \circlearrowleft Q_{1-s}$$

Hecke $Q \subset \mathbb{Q}$

$$\#Q_s^\chi = \sum_{n \geq 1} \frac{n^Q}{n^s} = \sum_{0 \neq n \in \mathbb{Z}} \frac{n^\chi}{n^s} = \prod_p \frac{p^s}{p^s - 1} \in \mathbb{C}^{\times} \setminus \{1\} \subset \mathbb{C}$$

$$\circlearrowleft Q_s = \overline{\det \omega_i^s} \left(\frac{\Gamma_{s/2}}{\pi^{s/2}} \right)^{r_{\mathbb{R}}} \left(\frac{\Gamma_s}{(2\pi)^s} \right)^{r_{\mathbb{C}}} \#Q_s$$

$$\circlearrowleft Q_s = \circlearrowleft Q_{1-s}$$