

${}^n\mathbb{C} \supset \mathcal{L} = \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} \cdot {}^{2n}\mathbb{Z}$ Gitter

$$\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} = \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_1 \cdots \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_{2n} \in {}^n\mathbb{C}_{2n}^{\mathbb{C}}$$

$$\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} = \left[\begin{array}{c|c} \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_1 & \cdots \\ \hline \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_1 & \cdots \end{array} \right] \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_{2n} \in {}^{2n}\mathbb{C}_{2n}^{\mathbb{C}}$$

Spalten $\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_1 \cdots \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_{2n}$ free $\Leftrightarrow \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_1 \cdots \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_{2n}$ free $_{\mathbb{C}}$

$${}^n\mathbb{C} + \underbrace{\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} \cdot {}^{2n}\mathbb{Z}}_{\text{hol}} \underset{\text{hol}}{\simeq} {}^n\mathbb{C} + \underbrace{\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} \cdot {}^{2n}\mathbb{Z}}_{\mathbb{C}} \Leftrightarrow \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} = \underbrace{\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}}_{\in {}^n\mathbb{C}_n^{\mathbb{C}}} \cdot \underbrace{\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}}_{\in {}^{2n}\mathbb{Z}_{2n}^{\mathbb{C}}}$$

${}^n\mathbb{C} + \underbrace{\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} \cdot {}^{2n}\mathbb{Z}}_{\text{hol}} \xleftarrow[\text{bihol}]{\mathcal{J}}$ ${}^n\mathbb{C} + \underbrace{\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} \cdot {}^{2n}\mathbb{Z}}_{\mathbb{C}}$ OE $\mathcal{J}_0 = 0 \Rightarrow$

$$\begin{array}{ccc} \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} \cdot {}^{2n}\mathbb{Z} & \xleftarrow{\mathcal{J}} & \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} \cdot {}^{2n}\mathbb{Z} \\ \cap & & \cap \\ {}^n\mathbb{C} & \xleftarrow{\mathcal{J}} & {}^n\mathbb{C} \\ \downarrow & & \downarrow \\ {}^n\mathbb{C} + \underbrace{\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} \cdot {}^{2n}\mathbb{Z}}_{\text{hol}} & \xleftarrow{\mathcal{J}} & {}^n\mathbb{C} + \underbrace{\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} \cdot {}^{2n}\mathbb{Z}}_{\mathbb{C}} \end{array}$$

$$\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} \cdot {}^{2n}\mathbb{Z} = \mathcal{J} \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} \cdot {}^{2n}\mathbb{Z} \Rightarrow \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_j \in \mathcal{J} \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} \cdot {}^{2n}\mathbb{Z} \Rightarrow \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_j = \mathcal{J} \underbrace{\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_j}_{\in {}^{2n}\mathbb{Z}} \Rightarrow \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} = \mathcal{J} \underbrace{\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}}_{\in {}^{2n}\mathbb{Z}} \curvearrowright \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} = (\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_1 \cdots \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_{2n}) \in {}^{2n}\mathbb{Z}_{2n}$$

$$\mathcal{J} \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_j \in \mathcal{J} \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} \cdot {}^{2n}\mathbb{Z} \Rightarrow \mathcal{J} \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_j = \mathcal{J} \underbrace{\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_j}_{\in {}^{2n}\mathbb{Z}} \Rightarrow \mathcal{J} \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} = \mathcal{J} \underbrace{\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}}_{\in {}^{2n}\mathbb{Z}} \curvearrowright \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} = (\begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_1 \cdots \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix}_{2n}) \in {}^{2n}\mathbb{Z}_{2n}$$

$$\begin{array}{c|c} \mathcal{J} & 0 \\ \hline 0 & \mathcal{J} \end{array} \begin{array}{c|c} \cdot \\ \vdots \\ \cdot \end{array} = \begin{array}{c|c} \mathcal{J} & \cdot \\ \hline \mathcal{J} & \cdot \end{array} = \frac{\begin{array}{c|c} \mathcal{J} & \cdot \\ \hline \mathcal{J} & \cdot \end{array}}{\begin{array}{c|c} \mathcal{J} & \cdot \\ \hline \mathcal{J} & \cdot \end{array}} = \frac{\begin{array}{c|c} \mathcal{J} & \cdot \\ \hline \mathcal{J} & \cdot \end{array}}{\begin{array}{c|c} \mathcal{J} & \cdot \\ \hline \mathcal{J} & \cdot \end{array}} = \frac{\begin{array}{c|c} \mathcal{J} & 0 \\ \hline 0 & \mathcal{J} \end{array}}{\begin{array}{c|c} \mathcal{J} & \cdot \\ \hline \mathcal{J} & \cdot \end{array}} \begin{array}{c|c} \cdot \\ \vdots \\ \cdot \end{array} \curvearrowright \begin{array}{c|c} \cdot \\ \vdots \\ \cdot \end{array} = {}^{2n}1_{2n} \Rightarrow \begin{bmatrix} \cdot \\ \vdots \\ \cdot \end{bmatrix} \in {}^{2n}\mathbb{Z}_{2n}^{\mathbb{C}}$$

$${}^n\mathbb{C} + \underbrace{\sqrt{-1}}_{\sqrt{-1}} \cdot {}^{2n}\mathbb{Z} \simeq {}^n\mathbb{C} + \underbrace{1 \cdot \sqrt{-1}}_{1 \cdot \sqrt{-1}} \cdot {}^{2n}\mathbb{Z} \curvearrowright \sqrt{-1} - \sqrt{-1} \in {}^n\mathbb{C}_n^{\mathbb{C}}$$

$${}_i\pi^j = \begin{array}{c|c|c|c} 1 & & & 0 \\ \hline & 0 & & 1 \\ \hline & & 1 & \\ \hline & 1 & & 0 \\ \hline 0 & & & 1 \end{array} \in {}^{2n}\mathbb{Z}_{2n}^{\mathbb{C}}$$

$$\mathbb{F} = \prod_i \pi^j \Rightarrow \sqrt{-1} \mathbb{F} = \underbrace{\sqrt{-1} \mid \mathbb{F}}_{\text{inv}} = \underbrace{\sqrt{-1} \mid \sqrt{-1}^{-1} \mathbb{F}}_{\sqrt{-1}^{-1} \mathbb{F} = \sqrt{-1}} \Rightarrow \sqrt{-1} = \underbrace{\sqrt{-1} \mid \sqrt{-1}}_{\sqrt{-1}} \mathbb{F}$$

$$\frac{\sqrt{-1}}{\sqrt{-1}} = \frac{\sqrt{-1} \mid 0}{0 \mid \sqrt{-1}} \frac{1 \mid \sqrt{-1}}{1 \mid \sqrt{-1}} \mathbb{F} \Rightarrow \frac{1 \mid \sqrt{-1}}{1 \mid \sqrt{-1}} \in {}^{2n}\mathbb{C}_{2n}^{\mathbb{C}}$$

$$\Rightarrow \frac{1 \mid \sqrt{-1}}{0 \mid \sqrt{-1} - \sqrt{-1}} = \frac{1 \mid 0}{-1 \mid 1} \frac{1 \mid \sqrt{-1}}{1 \mid \sqrt{-1}} \in {}^{2n}\mathbb{C}_{2n}^{\mathbb{C}} \Rightarrow \sqrt{-1} - \sqrt{-1} \in {}^n\mathbb{C}_n^{\mathbb{C}}$$

$${}^n\mathbb{C} + \underbrace{1 \cdot \sqrt{-1}}_{1 \cdot \sqrt{-1}} \cdot {}^{2n}\mathbb{Z} \simeq {}^n\mathbb{C} + \underbrace{1 \cdot \sqrt{-1}}_{1 \cdot \sqrt{-1}} \cdot {}^{2n}\mathbb{Z} \Leftrightarrow \begin{cases} \sqrt{-1} \mid \sqrt{-1} \in {}^{2n}\mathbb{Z}_{2n}^{\mathbb{C}} \\ \sqrt{-1} = \frac{-1}{\sqrt{-1} + \sqrt{-1}} \underbrace{\sqrt{-1} + \sqrt{-1}}_{\sqrt{-1} + \sqrt{-1}} \end{cases}$$

$$\underbrace{1 \mid \sqrt{-1}} = \underbrace{\sqrt{-1} \mid \sqrt{-1}}_{\sqrt{-1}} \frac{\sqrt{-1} \mid \sqrt{-1}}{\sqrt{-1} \mid \sqrt{-1}} = \underbrace{\sqrt{-1} + \sqrt{-1}}_{\sqrt{-1} + \sqrt{-1}} \mid \underbrace{\sqrt{-1} + \sqrt{-1}}_{\sqrt{-1} + \sqrt{-1}}$$

$$\Rightarrow 1 = \underbrace{\sqrt{-1} + \sqrt{-1}}_{\sqrt{-1} + \sqrt{-1}} \Rightarrow \sqrt{-1} = \frac{-1}{\sqrt{-1} + \sqrt{-1}} \Rightarrow \sqrt{-1} = \underbrace{\sqrt{-1} + \sqrt{-1}}_{\sqrt{-1} + \sqrt{-1}} = \frac{-1}{\sqrt{-1} + \sqrt{-1}} \underbrace{\sqrt{-1} + \sqrt{-1}}_{\sqrt{-1} + \sqrt{-1}}$$

$$\frac{\text{tori}}{\text{bihol}} = {}^n\mathbb{C}_n^{\mathbb{C}} \cap {}^n\mathbb{C}_{2n}^{\mathbb{C}} \cap {}^{2n}\mathbb{Z}_{2n}^{\mathbb{C}} = {}^n\mathbb{C}_n^{\mathcal{I} \text{ reg}} \cap {}^{2n}\mathbb{Z}_{2n}^{\mathbb{C}}$$

$$\frac{z \in \mathbb{C} : \mathcal{I} \neq 0}{\text{GL}_2(\mathbb{Z})} = \frac{z \in \mathbb{C} : \mathcal{I} z > 0}{\text{SL}_2(\mathbb{Z})} \xrightarrow{j} \mathbb{C}$$