$${}^{n}\mathbb{C} \sqsupset \mathcal{L} = \dot{\omega}_{\cdot}\, {}^{2n}\mathbb{Z} \text{ Gitter}$$

$$\dot{\omega}_{\cdot} = \dot{\omega}_{1} \cdots \dot{\omega}_{2n} \in {}^{n}\mathbb{C}^{\complement}_{2n}$$

$$\frac{\dot{\omega}_{\cdot}}{\dot{\bar{\omega}}_{\cdot}} = \frac{\dot{\omega}_{1} \;\Big|\; \cdots \;\Big|\; \dot{\omega}_{2n}}{\dot{\bar{\omega}}_{1} \;\Big|\; \cdots \;\Big|\; \dot{\bar{\omega}}_{2n}} \in {}^{2n}\mathbb{C}^{\complement}_{2n}$$

$$\text{Spalten } \dot{\omega}_{1} \cdots \dot{\omega}_{2n} \underset{\mathbb{R}}{\text{free}} \Leftrightarrow \frac{\dot{\omega}_{1}}{\dot{\bar{\omega}}_{1}} \cdots \frac{\dot{\omega}_{2n}}{\dot{\bar{\omega}}_{2n}} \underset{\mathbb{C}}{\text{free}}$$

$${}^{n}\mathbb{C} + \underbrace{\dot{\omega}_{\cdot}\,{}^{2n}\mathbb{Z}} \underset{\text{hol}}{\smile} {}^{n}\mathbb{C} + \underbrace{\dot{\tau}_{\cdot}\,{}^{2n}\mathbb{Z}} \Leftrightarrow \dot{\tau}_{\cdot} = \underbrace{\dot{\alpha}_{\cdot}}_{\in {}^{n}\mathbb{C}^{\complement}_{n}} \dot{\omega}_{\cdot} \underbrace{\dot{\mu}_{\cdot}}_{\in {}^{2n}\mathbb{Z}^{\complement}_{2n}}$$

$${}^{n}\mathbb{C} + \underbrace{\dot{\omega}_{\cdot}\,{}^{2n}\mathbb{Z}} \xleftarrow[\text{bihol}]{\mu} {}^{n}\mathbb{C} + \underbrace{\dot{\tau}_{\cdot}\,{}^{2n}\mathbb{Z}} \text{ OE } \mu_{0} = 0 \Longrightarrow$$

$$\begin{array}{ccc}
\dot{\omega}_{\cdot}\,{}^{2n}\mathbb{Z} & \xleftarrow[\alpha]{} & \dot{\tau}_{\cdot}\,{}^{2n}\mathbb{Z} \\
\cap & & \cap \\
{}^{n}\mathbb{C} & \xleftarrow[\alpha]{} & {}^{n}\mathbb{C} \\
\downarrow & & \downarrow \\
{}^{n}\mathbb{C} + \underbrace{\dot{\omega}_{\cdot}\,{}^{2n}\mathbb{Z}} & \xleftarrow[\mu]{} & {}^{n}\mathbb{C} + \underbrace{\dot{\tau}_{\cdot}\,{}^{2n}\mathbb{Z}}
\end{array}$$

$$\tau\,{}^{2n}\mathbb{Z} = \alpha\,\omega\,{}^{2n}\mathbb{Z} \Longrightarrow \tau_{j} \in \alpha\,\omega\,{}^{2n}\mathbb{Z} \Longrightarrow \tau_{j} = \alpha\,\omega \underbrace{\mu_{j}}_{\in {}^{2n}\mathbb{Z}} \Longrightarrow \tau_{\cdot} = \alpha\,\omega\,\mu_{\cdot} \curvearrowleft \mu_{\cdot} = (\mu_{1} \cdots \mu_{2n}) \in {}^{2n}\mathbb{Z}_{2n}$$

$$\alpha\,\omega_{j} \in \tau\,{}^{2n}\mathbb{Z} \Longrightarrow \alpha\,\omega_{j} = \tau \underbrace{\nu_{j}}_{\in {}^{2n}\mathbb{Z}} \Longrightarrow \alpha\,\omega_{\cdot} = \tau\,\nu_{\cdot} \curvearrowleft \nu_{\cdot} = (\nu_{1} \cdots \nu_{2n}) \in {}^{2n}\mathbb{Z}_{2n}$$

$$\frac{\alpha \;\Big|\; 0}{0 \;\Big|\; \bar{\alpha}}\;\frac{\omega}{\bar{\omega}} = \frac{\alpha \;\Big|\; \omega}{\bar{\alpha} \;\Big|\; \bar{\omega}} = \frac{\tau\,\nu}{\bar{\tau}\,\nu} = \frac{\tau}{\bar{\tau}}\,\nu = \frac{\alpha\,\omega\,\mu}{\bar{\alpha}\,\bar{\omega}\,\mu}\,\nu = \frac{\alpha \;\Big|\; 0}{0 \;\Big|\; \bar{\alpha}}\;\frac{\omega}{\bar{\omega}}\,\mu\,\nu \Longrightarrow \mu\,\nu = {}^{2n}1_{2n} \Longrightarrow \mu \in {}^{2n}\mathbb{Z}^{\complement}_{2n}$$

$$^{n}\mathbb{C}+\underbrace{\tau}\,{}^{2n}\mathbb{Z}\asymp{}^{n}\mathbb{C}+\underbrace{\overbrace{1:\tau}}\,{}^{2n}\mathbb{Z}\curvearrowleft\bar{\tau}-\tau\in{}^{n}\mathbb{C}_{n}^{\text{Im}}$$

$$_{i}\pi^{j}=\begin{array}{c|c|c|c|c}1&&&&0\\\hline &0&&1&\\\hline &&1&&\\\hline &1&&0&\\\hline 0&&&&1\end{array}\in{}^{2n}\mathbb{Z}_{2n}^{\complement}$$

$$\sigma=\prod{}_{i}\pi^{j}\Rightarrow\tau\sigma=\underset{\text{inv}}{\omega}\,\Big|\,\sigma=\omega\underbrace{1\,\Big|\,\omega^{-1}\sigma}\underset{\omega^{-1}\sigma=\tau}{\Rightarrow}\tau=\omega\underbrace{1\,\Big|\,\tau}\sigma$$

$$\frac{\tau}{\bar{\tau}}=\begin{array}{c|c}\omega&0\\\hline 0&\bar{\omega}\end{array}\begin{array}{c|c}1&\tau\\\hline 1&\bar{\tau}\end{array}\sigma\Rightarrow\begin{array}{c|c}1&\tau\\\hline 1&\bar{\tau}\end{array}\in{}^{2n}\mathbb{C}_{2n}^{\complement}$$

$$\Rightarrow\begin{array}{c|c}1&\tau\\\hline 0&\bar{\tau}-\tau\end{array}=\begin{array}{c|c}1&0\\\hline -1&1\end{array}\begin{array}{c|c}1&\tau\\\hline 1&\bar{\tau}\end{array}\in{}^{2n}\mathbb{C}_{2n}^{\complement}\Rightarrow\bar{\tau}-\tau\in{}^{n}\mathbb{C}_{n}^{\complement}$$

$$^{n}\mathbb{C}+\underbrace{\overbrace{1:\tau}}\,{}^{2n}\mathbb{Z}\asymp{}^{n}\mathbb{C}+\underbrace{\overbrace{1:\tau'}}\,{}^{2n}\mathbb{Z}\Leftrightarrow\begin{cases}\bigvee\quad\begin{array}{c|c}\alpha&\beta\\\hline\gamma&\delta\end{array}\in{}^{2n}\mathbb{Z}_{2n}^{\complement}\\ \tau'=\overbrace{\alpha+\tau\gamma}^{-1}\underbrace{\beta+\tau\delta}\end{cases}$$

$$\underbrace{1\,\big|\,\tau'}=\rho\underbrace{1\,\big|\,\tau}\begin{array}{c|c}\alpha&\beta\\\hline\gamma&\delta\end{array}=\rho\underbrace{\alpha+\tau\gamma}\,\Big|\,\rho\underbrace{\beta+\tau\delta}$$

$$\Rightarrow 1=\rho\underbrace{\alpha+\tau\gamma}\Rightarrow\rho=\overbrace{\alpha+\tau\gamma}^{-1}\Rightarrow\tau'=\rho\underbrace{\beta+\tau\delta}=\overbrace{\alpha+\tau\gamma}^{-1}\underbrace{\beta+\tau\delta}$$

$$\frac{\text{tori}}{\text{bihol}}={}^{n}\mathbb{C}_{n}^{\complement}\sqcap{}^{n}\mathbb{C}_{2n}^{\complement}\urcorner{}^{2n}\mathbb{Z}_{2n}^{\complement}={}^{n}\mathbb{C}_{n}^{\mathcal{I}\text{ reg}}\urcorner{}^{2n}\mathbb{Z}_{2n}^{\complement}$$

$$\frac{z\in\mathbb{C}:\mathcal{I}\neq 0}{\mathrm{GL}_{2}(\mathbb{Z})}=\frac{z\in\mathbb{C}:\mathcal{I}z>0}{\mathrm{SL}_{2}(\mathbb{Z})}\xrightarrow[\asymp]{j}\mathbb{C}$$