

$$\frac{\Gamma_{1+a/2}^r}{\Gamma_{1+ra/2}} \int_{\mathbb{R}^r} x \mathbf{e}_x^{-1/2} \prod_{i < j} \overline{x_j - x_i}^a = \frac{\Gamma_{d/r}^X}{(2\pi)^{d/2-r}}$$

$$\begin{aligned} \text{LHS} & \stackrel{\text{FK}_{121}}{=} \frac{\sqrt{2\pi}^r}{\Gamma_{1+ra/2}} \prod_{1 \leq k \leq r} \Gamma_{1+ka/2} = \sqrt{2\pi}^r \prod_{1 \leq j \leq r} \Gamma_{1+(r-j)a/2} \\ & = \sqrt{2\pi}^r \prod_{1 \leq j \leq r} \Gamma_{d/r - (j-1)a/2} = \sqrt{2\pi}^r \frac{\Gamma_{d/r}^X}{\sqrt{2\pi}^{d-r}} = \text{RHS} \end{aligned}$$

$$\int_{dx/\sqrt{2\pi}^d}^X x \mathbf{e}_x^{-A/2} \stackrel{\text{FK}_{350}}{=} \frac{-1/2}{\sqrt{A}} : \int_{dx/\sqrt{2\pi}^d}^X x \mathbf{e}_x^{-1/2} = 1 : \int_{dx}^X x \mathbf{e}_x^{-1/2} = \sqrt{2\pi}^d$$

$$\overrightarrow{\mathbb{R}}^r : x_1 < \dots < x_r$$

$$\frac{\Gamma_{d/r}^X}{(2\pi)^{d-r}} \int_{dx}^X x \eta = \frac{\Gamma_{1+a/2}^r}{\Gamma_{1+ra/2}} \int_{\mathbb{R}^r} \prod_{i < j} \overline{x_j - x_i}^a \int_{dh}^{\text{Aut } X} x \cdot h \eta$$

$$\int_{dx}^X x \eta \stackrel{\text{FK}_{104}}{=} c_0 \int_{\overrightarrow{\mathbb{R}}^r} \prod_{i < j} \overline{x_j - x_i}^a \int_{dh}^{\text{Aut } X} x \cdot h \eta = \frac{c_0}{r!} \int_{\mathbb{R}^r} \prod_{i < j} \overline{x_j - x_i}^a \int_{dh}^{\text{Aut } X} x \cdot h \eta$$

$$x \eta = -x \mathbf{e}_x^{-1/2} \Rightarrow \text{LHS} = \frac{\Gamma_{d/r}^X}{(2\pi)^{d-r}} \sqrt{2\pi}^d = \frac{\Gamma_{d/r}^X}{(2\pi)^{d/2-r}} = \text{RHS}$$