

$$\begin{aligned} x \square_{\underline{\quad}}^{\mu} &= x \mathfrak{e}^{\mu} x \square_{\underline{\quad}} + \underbrace{x \square_{\underline{\quad}} - x \mathfrak{e}^{\nu} x_{\nu} \square_{\underline{\quad}}}_{\square_{\underline{\quad}}^{\mu}} x \square_{\underline{\quad}}^{\mu} = x \mathfrak{e}^{\nu} \overbrace{\nu \delta^{\mu} x \square_{\underline{\quad}} - x_{\nu} x \square_{\underline{\quad}}^{\mu}}^{\square_{\underline{\quad}}^{\mu}} + x \square_{\underline{\quad}} x \square_{\underline{\quad}}^{\mu} \\ \square_{\underline{\quad}}^{\mu} &= \mathfrak{e}^{\mu} \square_{\underline{\quad}} + \underbrace{\square_{\underline{\quad}} - \mathfrak{e}^{\nu} x_{\nu} \square_{\underline{\quad}}}_{\square_{\underline{\quad}}^{\mu}} = \mathfrak{e}^{\nu} \overbrace{\nu \delta^{\mu} \square_{\underline{\quad}} - x_{\nu} \square_{\underline{\quad}}^{\mu}}^{\square_{\underline{\quad}}^{\mu}} + \square_{\underline{\quad}} \square_{\underline{\quad}}^{\mu} \end{aligned}$$

$$\square_{\underline{\quad}}^{\mu} \text{ conserved } 0 \\ \text{el current}$$

$$\begin{aligned} \text{LHS} &= \underbrace{x \mathfrak{e}^{\nu} \nu \delta^{\mu} \square_{\underline{\quad}} - x_{\nu} \square_{\underline{\quad}}^{\mu}}_{\square_{\underline{\quad}}^{\mu}} + \square_{\underline{\quad}} \square_{\underline{\quad}}^{\mu} \\ &= \underbrace{x_{\mu} \mathfrak{e}^{\nu} \nu \delta^{\mu} x \square_{\underline{\quad}} - x_{\nu} x \square_{\underline{\quad}}^{\mu}}_{\square_{\underline{\quad}}^{\mu}} + \underbrace{x \mathfrak{e}^{\nu} \nu \delta^{\mu} \square_{\underline{\quad}} - x_{\nu} \square_{\underline{\quad}}^{\mu}}_{\square_{\underline{\quad}}^{\mu}} + \underbrace{x \square_{\underline{\quad}} x \square_{\underline{\quad}}^{\mu}}_{\square_{\underline{\quad}}^{\mu}} + \underbrace{x \square_{\underline{\quad}} x \square_{\underline{\quad}}^{\mu}}_{\square_{\underline{\quad}}^{\mu}} \\ &\stackrel{\text{harm}}{=} \underbrace{x_{\mu} \mathfrak{e}^{\nu} \nu \delta^{\mu} x \square_{\underline{\quad}} - x_{\nu} x \square_{\underline{\quad}}^{\mu}}_{\square_{\underline{\quad}}^{\mu}} + \underbrace{x \mathfrak{e}^{\nu} x \square_{\underline{\quad}}}_{\square_{\underline{\quad}}^{\mu}} + \underbrace{x \square_{\underline{\quad}} x \square_{\underline{\quad}}^{\mu}}_{\square_{\underline{\quad}}^{\mu}} + \underbrace{x \square_{\underline{\quad}} x \square_{\underline{\quad}}^{\mu}}_{\square_{\underline{\quad}}^{\mu}} \\ &= \underbrace{x_{\mu} \mathfrak{e}^{\mu} x \square_{\underline{\quad}}}_{\square_{\underline{\quad}}^{\mu}} + \underbrace{x \mathfrak{e}^{\nu} x \square_{\underline{\quad}}}_{\square_{\underline{\quad}}^{\mu}} + \underbrace{x \square_{\underline{\quad}} x \square_{\underline{\quad}}^{\mu}}_{\square_{\underline{\quad}}^{\mu}} + \underbrace{x \square_{\underline{\quad}} x \square_{\underline{\quad}}^{\mu}}_{\square_{\underline{\quad}}^{\mu}} - \underbrace{x_{\mu} \mathfrak{e}^{\nu} x_{\nu} x \square_{\underline{\quad}}^{\mu}}_{\square_{\underline{\quad}}^{\mu}} \\ &= \underbrace{x_{\mu} \mathfrak{e}^{\mu} \square_{\underline{\quad}} x \square_{\underline{\quad}}}_{\square_{\underline{\quad}}^{\mu}} + \underbrace{x \mathfrak{e}^{\nu} \square_{\underline{\quad}} x \square_{\underline{\quad}}}_{\square_{\underline{\quad}}^{\mu}} + \underbrace{x \square_{\underline{\quad}} x \square_{\underline{\quad}} x \square_{\underline{\quad}}}_{\square_{\underline{\quad}}^{\mu}} + \underbrace{x \square_{\underline{\quad}} x \square_{\underline{\quad}} x \square_{\underline{\quad}}}_{\square_{\underline{\quad}}^{\mu}} - \underbrace{x_{\mu} \mathfrak{e}^{\nu} x_{\nu} x \square_{\underline{\quad}}^{\mu}}_{\square_{\underline{\quad}}^{\mu}} \stackrel{\text{Lie alg inv}}{=} 0 \end{aligned}$$

$$\text{conserved el charge } \partial_t \int_S \square_{\underline{\quad}}^0 = 0$$

$$\begin{aligned} 0 &= \partial_{\mu} \square_{\underline{\quad}}^{\mu} = \mathfrak{e} \cdot \square_{\underline{\quad}} + \partial_t \square_{\underline{\quad}}^0 \\ \Rightarrow 0 &= \int_S \partial_{\mu} \square_{\underline{\quad}}^{\mu} = \underbrace{\int_S \mathfrak{e} \cdot \square_{\underline{\quad}}}_{=0} + \int_S \partial_t \square_{\underline{\quad}}^0 = \partial_t \int_S \square_{\underline{\quad}}^0 \end{aligned}$$

$$\text{Poincare current } \mathcal{J}_{\underline{\quad}; \underline{\quad}}^{\mu} = \underbrace{x^{\nu} \mathcal{L}^{\mu} + \mathcal{L}^{\mu} x}_{\mathcal{L}_{\mathcal{N}; \mathcal{N}}} - \eta^{\mu\lambda} \lambda \mathcal{N} \left(x^{\mu} \mathcal{L}^{\nu} + \mathcal{L}^{\nu} \mathcal{N} + \mathcal{N} \right)$$