

$$\begin{aligned} \left\{ \begin{array}{c} \mathbb{N} \\ \mu \mathbb{N} \end{array} \right\} &\stackrel{\text{group}}{\underset{\text{inv}}{=}} \det x_{\underline{\mathcal{L}}} \left\{ \begin{array}{c} \mathbb{N} \\ x_{\mu}^{-1\nu} \overbrace{\left[ \begin{array}{c} \mathbb{N} \\ \nu \mathbb{N} \end{array} \right]} \end{array} \right\} \\ \mathbb{N} : \mu \mathbb{N} &\stackrel{\text{group}}{\underset{\text{inv}}{=}} \det x_{\underline{\mathcal{L}}} \overbrace{\left[ \begin{array}{c} \mathbb{N} : x_{\mu}^{-1\nu} \overbrace{\left[ \begin{array}{c} \mathbb{N} \\ \nu \mathbb{N} \end{array} \right]} \end{array} \right]} \end{aligned}$$

$$0 \stackrel{\text{Lie alg}}{\underset{\text{inv}}{=}} x_{\mu} \mathfrak{L}^{\mu} \mathbb{N} : \mathbb{N} + x_{\nu} \mathfrak{L}^{\nu} \overbrace{\left[ \begin{array}{c} \mathbb{N} \\ \nu \mathbb{N} \end{array} \right]} + \overbrace{\left[ \begin{array}{c} \mathbb{N} \\ \mathbb{N} : \mathbb{N} \end{array} \right]} + \overbrace{\left[ \begin{array}{c} \mathbb{N} \\ \mu \mathbb{N} \end{array} \right]} + \overbrace{\left[ \begin{array}{c} \mathbb{N} \\ \nu \mathbb{N} \end{array} \right]} - x_{\mu} \mathfrak{L}^{\nu} \overbrace{\left[ \begin{array}{c} \mathbb{N} \\ \nu \mathbb{N} \end{array} \right]}^{\mu}$$

$$\mathbb{N} = \partial_t \overline{\mathbb{N}} t$$

$$x_{\mathfrak{L}} = \partial_t x_{\mathfrak{G}} \Rightarrow \partial_t \det x_{\underline{\mathfrak{G}}} = \text{tr } \partial_t x_{\underline{\mathfrak{G}}} = \text{tr } x_{\underline{\mathfrak{L}}} = x_{\mu} \mathfrak{L}^{\mu}$$

$$\begin{aligned} 0 = \partial_t \overline{\mathbb{N} : \mathbb{N}} &= \partial_t \det x_{\underline{\mathfrak{G}}} \overbrace{\left[ \begin{array}{c} \mathbb{N} : x_{\mu}^{-1\nu} \overbrace{\left[ \begin{array}{c} \mathbb{N} \\ \nu \mathbb{N} \end{array} \right]} t + \nu \mathbb{N} \overline{\mathbb{N}} t \end{array} \right]} = \partial_t \det x_{\underline{\mathfrak{G}}} \overline{\mathbb{N} : \mathbb{N}} + \partial_t \overbrace{\left[ \begin{array}{c} \mathbb{N} : x_{\mu}^{-1\nu} \overbrace{\left[ \begin{array}{c} \mathbb{N} \\ \nu \mathbb{N} \end{array} \right]} t + \nu \mathbb{N} \overline{\mathbb{N}} t \end{array} \right]} \\ &= x_{\mu} \mathfrak{L}^{\mu} \overline{\mathbb{N} : \mathbb{N}} + x_{\nu} \mathfrak{L}^{\nu} \overbrace{\left[ \begin{array}{c} \mathbb{N} \\ \nu \mathbb{N} \end{array} \right]} + \overbrace{\left[ \begin{array}{c} \mathbb{N} \\ \mathbb{N} : \mathbb{N} \end{array} \right]} + \overbrace{\left[ \begin{array}{c} \mathbb{N} \\ \mu \mathbb{N} \end{array} \right]} + \overbrace{\left[ \begin{array}{c} \mathbb{N} \\ \nu \mathbb{N} \end{array} \right]} - x_{\mu} \mathfrak{L}^{\nu} \overbrace{\left[ \begin{array}{c} \mathbb{N} \\ \nu \mathbb{N} \end{array} \right]}^{\mu} = \text{RHS} \end{aligned}$$

$$x^{\nu} : \mu \mathbb{N} \in \mathbb{R}^d \times \mathbb{R} \times \mathbb{R} \xrightarrow[\text{el current}]{\overline{\mathbb{N} : \mathbb{N}}^{\mu}} \mathbb{R} \ni \overline{\mathbb{N} : \mathbb{N}}^{\mu}$$

$$\overline{\mathbb{N} : \mathbb{N}}^{\mu} = x_{\mathfrak{L}}^{\mu} \overline{\mathbb{N} : \mathbb{N}} + \underbrace{\overline{\mathbb{N}} - x_{\nu} \mathfrak{L}^{\nu} \overbrace{\left[ \begin{array}{c} \mathbb{N} \\ \nu \mathbb{N} \end{array} \right]}^{\mu}}_{\text{gauge symmetries}} = x_{\nu} \mathfrak{L}^{\nu} \overbrace{\left[ \begin{array}{c} \mathbb{N} \\ \nu \mathfrak{L}^{\mu} \mathbb{N} - \nu \mathbb{N} \overline{\mathbb{N} : \mathbb{N}}^{\mu} \end{array} \right]} + \overline{\mathbb{N}} \overline{\mathbb{N} : \mathbb{N}}^{\mu}$$

gauge symmetries

$$g \cdot \varphi$$