$$ {}^{x}\underset{0}{\boxed{Ч:}} = \underset{0}{\boxed{x{:}\,{}^{x}Ч{:}\,{}^{x}_{\mu}\underline{Ч}}} = -\underline{\mathcal{V}}_{{}^{x}Ч} $$

$$ {}^{x}\underset{0}{\boxed{Ч:}}^{\mu} = {}^{x}\underset{0}{\boxed{x{:}\,{}^{x}Ч{:}\,{}^{x}_{\mu}\underline{Ч}}}^{\mu} = \eta^{\mu\nu}\,{}^{x}_{\nu}\underline{Ч} $$

$$ \underset{0}{\boxed{Ч:}} \underset{\text{motion}}{=} \underline{{}_{\mu}\underset{0}{\boxed{Ч:}}^{\mu}} $$

$$ Ч \in \mathbb{R}^{d} \overset{0}{\underset{\infty}{\to}} \mathbb{R} \text{ vanish at } \infty \Longrightarrow \int_{dx} {}^{x}_{\mu}\underline{Ч}\,{}^{x}\mathrm{ʔ} = -\int_{dx} {}^{x}Ч\,{}^{x}_{\mu}\underline{\mathrm{ʔ}} $$

$$ Ч\,\underline{{}_{Ч}{}^{x}\mathcal{L}} = {}^{x}Ч\,\underset{0}{\boxed{x{:}\,{}^{x}Ч{:}\,{}^{x}\underline{Ч}}} + {}^{x}_{\mu}\underline{Ч}\,\underset{0}{\boxed{x{:}\,{}^{x}Ч{:}\,{}^{x}\underline{Ч}}}^{\mu} = {}^{x}Ч\,{}^{x}\underset{0}{\boxed{Ч:}} + {}^{x}_{\mu}\underline{Ч}\,{}^{x}\underset{0}{\boxed{Ч:}}^{\mu} \Longrightarrow $$

$$ Ч\,\underline{{}_{Ч}\int_{dx}{}^{x}\mathcal{L}} = \int_{dx} Ч\,\underline{{}_{Ч}{}^{x}\mathcal{L}} = \int_{dx} {}^{x}Ч\,{}^{x}\underset{0}{\boxed{Ч:}} + {}^{x}_{\mu}\underline{Ч}\,{}^{x}\underset{0}{\boxed{Ч:}}^{\mu} = \int_{dx} {}^{x}Ч\,{}^{x}\underset{0}{\boxed{Ч:}} - {}^{x}Ч\,\underline{{}_{\mu}{}^{x}\underset{0}{\boxed{Ч:}}^{\mu}} = \int_{dx} {}^{x}Ч\,{}^{x}\overbrace{\underset{0}{\boxed{Ч:}} - \underline{{}_{\mu}\underset{0}{\boxed{Ч:}}^{\mu}}}^{=0} $$

$$ \underline{{}_{\mu}\,{}_{\nu}\delta^{\mu}\boxed{Ч:} - {}_{\nu}\underline{Ч}\,\underset{0}{\boxed{Ч:}}^{\mu}} = {}_{\nu}\boxed{Ч:} $$

$$ \underline{{}_{\nu}\boxed{Ч:}} = {}_{\nu}\boxed{Ч:} + {}_{\nu}\underline{Ч}\,\underset{0}{\boxed{Ч:}} + {}_{\nu\mu}\underline{\underline{Ч}}\,\underset{0}{\boxed{Ч:}}^{\mu} $$

$$ \Longrightarrow \text{LHS} = \underline{{}_{\nu}\boxed{Ч:}} - \overbrace{{}_{\mu\nu}\underline{\underline{Ч}}\,\underset{0}{\boxed{Ч:}}^{\mu} + \underline{{}_{\mu}\,{}_{\nu}\underline{Ч}\,\underset{0}{\boxed{Ч:}}^{\mu}}_{*}} $$

$$ = {}_{\nu}\boxed{Ч:} + {}_{\nu}\underline{Ч}\,\underset{0}{\boxed{Ч:}} + {}_{\nu\mu}\underline{\underline{Ч}}\,\underset{0}{\boxed{Ч:}}^{\mu} - {}_{\mu\nu}\underline{\underline{Ч}}\,\underset{0}{\boxed{Ч:}}^{\mu} - {}_{\nu}\underline{Ч}\,\underset{0}{\overset{*}{\boxed{Ч:}}} = {}_{\nu}\boxed{Ч:} $$

$$ \text{motion}\ \eta^{\mu\nu}\,{}^{x}_{\mu\nu}\underline{\underline{Ч}} = -\underline{\mathcal{V}}_{{}^{x}Ч} $$

$$ -\underline{\mathcal{V}}_{{}^{x}Ч} = {}^{x}\underset{0}{\boxed{Ч:}} \underset{\text{motion}}{=} \underline{{}_{\mu}{}^{x}\underset{0}{\boxed{Ч:}}^{\mu}} = {}^{x}\underline{\eta^{\mu\nu}\,{}_{\nu}\underline{Ч}} = \eta^{\mu\nu}\,{}^{x}_{\mu\nu}\underline{\underline{Ч}} $$

constant solutions

$$\underline{\mathcal{V}}_v = 0$$

$$\underline{\underline{\mathcal{V}}}_v = m^2 \Longrightarrow \text{ min=vac}$$