

$$\begin{aligned}
& \mathbb{R}^d \xrightarrow[\text{fields}]{\mathcal{N}} \mathbb{R} \ni x_{\mathcal{N}} \\
{}^x \square{\mathcal{N}} = & \square{{}^x \mathcal{N} : x_{\underline{\mu}}} = \begin{cases} {}^x \mathcal{N} \\ x_{\underline{\mu}} \end{cases} = \frac{1}{2} \eta^{\mu\nu} x_{\underline{\mu}} x_{\underline{\nu}} - \mathcal{V}_{x_{\mathcal{N}}} \\
& \mathbb{R}^{1:d} \xrightarrow{\varphi} {}^N \mathbb{R} \\
\{\varphi\} = & \frac{\partial_{\underline{\mu}} \varphi \eta^{\mu\nu} \partial_{\underline{\nu}} \varphi}{2} - \mathcal{V}(\varphi) \\
& \text{constant solutions} \\
& \underline{\mathcal{V}}_v = 0 \\
& \underline{\mathcal{V}}_v = m^2 \Rightarrow \text{min=vac} \\
& \varphi \\
& d\varphi \\
\mathcal{L}_0(\varphi\dot{\varphi}) = & \frac{\dot{\varphi} \star \dot{\varphi} - m^2 \varphi \star \varphi}{2} \\
\mathcal{L}_0(\varphi) = & \frac{d\varphi \star d\varphi - m^2 \varphi \star \varphi = \partial\varphi \star \partial\varphi - m^2 \varphi^2}{2} \\
\mathcal{L}_0(\varphi\dot{\varphi}) = & \frac{\dot{\varphi} \star \dot{\varphi} - m^2 \varphi^2}{2} + g \frac{\varphi^4}{4!} \\
\mathcal{L}_0(\varphi) = & \frac{d\varphi \star d\varphi - m^2 \varphi \star \varphi}{2} = \frac{\partial\varphi \star \partial\varphi - m^2 \varphi^2}{2} + g \frac{\varphi^4}{4!} \\
& \dot{\varphi} \star \dot{\varphi} = \dot{\varphi}_{\underline{\mu}} g^{\mu\nu}(\varphi) \dot{\varphi}_{\underline{\nu}} \\
\mathcal{L}_0(\varphi\dot{\varphi}_{\underline{\mu}}) = & \frac{\dot{\varphi}_{\underline{\mu}} g^{\mu\nu}(\varphi) \dot{\varphi}_{\underline{\nu}} - m^2 \varphi \star \varphi}{2} \\
& d\varphi = \partial_{\underline{\mu}} \varphi dx^{\underline{\mu}} = {}_{\underline{\mu}} \mathbb{1} \varphi \mathbb{1}^{\underline{\mu}} \\
& d\varphi \star d\varphi = \partial_{\underline{\mu}} \varphi g^{\mu\nu} \varphi \partial_{\underline{\nu}} \varphi = {}_{\underline{\mu}} \mathbb{1} \varphi g^{\mu\nu} \varphi {}_{\underline{\nu}} \mathbb{1} \varphi \\
\mathcal{L}_0(\varphi) = & \frac{\partial_{\underline{\mu}} \varphi g^{\mu\nu} \varphi \partial_{\underline{\nu}} \varphi - m^2 \varphi^2}{2} = \frac{{}_{\underline{\mu}} \mathbb{1} \varphi g^{\mu\nu} \varphi {}_{\underline{\nu}} \mathbb{1} \varphi - m^2 \varphi^2}{2}
\end{aligned}$$