

$$\Omega = \partial\gamma \in \underbrace{\mathfrak{h} \times \mathfrak{h}}_{\mathfrak{h}} \times \underbrace{\mathfrak{h} \times \mathfrak{h}}_{\mathfrak{h}} \times \mathbb{R}^{2,d}$$

$$\omega = \int^{\mathfrak{h}_0} \Omega \in \underbrace{\mathfrak{h} \times \mathfrak{h}}_{\mathfrak{h}} \times \underbrace{\mathfrak{h} \times \mathfrak{h}}_{\mathfrak{h}} \times \mathbb{R}^2 \text{ sympl}$$

$$T \times \underbrace{\mathbb{L}}_{\mathfrak{h}} \xrightarrow[t^i: q_\alpha^j: |\alpha| \leq k]{\text{coord}} \mathbb{R}$$

$$\psi q_\alpha^j = \partial_\alpha (\psi \times q^j)$$

$$dt^i = Dt^i$$

$$\delta t^i = 0$$

$$\delta q_\alpha^j = \delta q_\alpha^j$$

$$Dq_\alpha^j = \begin{cases} dt^i q_{\alpha+\varepsilon_i}^j & |\alpha| < k \\ 0 & |\alpha| = k \end{cases}$$

$$DF = dt^i \left(\frac{\partial F}{\partial t^i} + \frac{\partial F}{\partial q_\alpha^j} q_{\alpha+\varepsilon_i}^j \right)$$

$$\delta F = \frac{\partial F}{\partial q_\alpha^j} \delta q_\alpha^j$$

$$dF = \frac{\partial F}{\partial t^i} dt^i + \frac{\partial F}{\partial q_\alpha^j} dq_\alpha^j = \frac{\partial F}{\partial t^i} dt^i + \frac{\partial F}{\partial q_\alpha^j} \underbrace{Dq_\alpha^j + \delta q_\alpha^j}_{=} = \frac{\partial F}{\partial t^i} dt^i + \frac{\partial F}{\partial q_\alpha^j} \underbrace{dt^i q_{\alpha+\varepsilon_i}^j + \delta q_\alpha^j}_{=}$$

$$= \underbrace{dt^i \frac{\partial F}{\partial t^i} + \frac{\partial F}{\partial q_\alpha^j} q_{\alpha+\varepsilon_i}^j}_{=DF} + \underbrace{\frac{\partial F}{\partial q_\alpha^j} \delta q_\alpha^j}_{=\delta F}$$

$$\delta \underline{\mathcal{L}} dt = \underset{\text{source}}{E} + D \underset{0,1}{M} \Rightarrow M = -\frac{\partial \mathcal{L}}{\partial \dot{q}^j} \delta q^j$$

$$E = \underbrace{\frac{\partial \mathcal{L}}{\partial q^j} - \partial_t \frac{\partial \mathcal{L}}{\partial \dot{q}^j}} \delta q^j \wedge dt$$

$$\begin{aligned} D \left(\frac{\partial \mathcal{L}}{\partial \dot{q}^j} \delta q^j \right) &= D \frac{\partial \mathcal{L}}{\partial \dot{q}^j} \times \delta q^j + \frac{\partial \mathcal{L}}{\partial \dot{q}^j} D \delta q^j = \partial_t \frac{\partial \mathcal{L}}{\partial \dot{q}^j} dt \times \delta q^j - \frac{\partial \mathcal{L}}{\partial \dot{q}^j} \delta D q^j \\ &= \partial_t \frac{\partial \mathcal{L}}{\partial \dot{q}^j} dt \times \delta q^j - \frac{\partial \mathcal{L}}{\partial \dot{q}^j} \delta \underbrace{dt \times \dot{q}^j} = \partial_t \frac{\partial \mathcal{L}}{\partial \dot{q}^j} dt \times \delta q^j - \frac{\partial \mathcal{L}}{\partial \dot{q}^j} dt \times \delta \dot{q}^j \\ \delta \underline{\mathcal{L}} dt &= d \underline{\mathcal{L}} dt = \underline{\mathcal{L}} \times dt = \left(\frac{\partial \mathcal{L}}{\partial t} dt + \frac{\partial \mathcal{L}}{\partial q^j} dq^j + \frac{\partial \mathcal{L}}{\partial \dot{q}^j} d\dot{q}^j \right) \times dt \\ &= \overbrace{\frac{\partial \mathcal{L}}{\partial q^j} D q^j + \delta q^j} + \overbrace{\frac{\partial \mathcal{L}}{\partial \dot{q}^j} D \dot{q}^j + \delta \dot{q}^j} \times dt = \overbrace{\frac{\partial \mathcal{L}}{\partial q^j} dt \dot{q}^j + \delta q^j} + \overbrace{\frac{\partial \mathcal{L}}{\partial \dot{q}^j} \delta \dot{q}^j} \times dt \\ &\Rightarrow \delta \underline{\mathcal{L}} dt + D \left(\frac{\partial \mathcal{L}}{\partial \dot{q}^j} \delta q^j \right) = \underbrace{\frac{\partial \mathcal{L}}{\partial q^j} - \partial_t \frac{\partial \mathcal{L}}{\partial \dot{q}^j}} \delta q^j \wedge dt \end{aligned}$$