

$$\mathfrak{I} \cdot \mathfrak{K} \in \begin{matrix} 2 \\ 0 \end{matrix} \mathbb{K} \text{ HilbR} \Leftrightarrow \begin{cases} \mathfrak{I} \mathfrak{K}^* = \mathfrak{I} \mathfrak{K} \\ \mathfrak{I} \mathfrak{K} \geq 0 \\ \mathfrak{I} \mathfrak{K} = 0 \end{cases} \Rightarrow \mathfrak{I} = 0 \quad \text{treu}$$

$$\mathfrak{I}^{\dagger} = \frac{1 \in \mathfrak{I}}{\mathfrak{I} \mathfrak{K} = 0: \bigwedge_{\mathfrak{I}} \mathfrak{I} \mathfrak{K} = 0} \text{ radical}$$

$$\mathfrak{I} \mathfrak{K} = 0 \Rightarrow \mathfrak{I} \mathfrak{K}^* = 0$$

$$\mathbb{R} \ni t \xrightarrow{\text{diff}} {}^t \mathfrak{L} = \underbrace{(1+t) \mathfrak{K} (1+t)} = \mathfrak{I} \mathfrak{K} + t \underbrace{(\mathfrak{I} \mathfrak{K} + \mathfrak{I} \mathfrak{K}^*)} + t^2 \mathfrak{I} \mathfrak{K} \geq 0 = \mathfrak{I} \mathfrak{K} = {}^0 \mathfrak{L} \text{ min}$$

$$\Rightarrow 0 = {}^0 \mathfrak{L} = \mathfrak{I} \mathfrak{K} + \mathfrak{I} \mathfrak{K}^* = 2 \Re \mathfrak{I} \mathfrak{K}$$

$$\begin{cases} \mathbb{K} = \mathbb{R} \\ \mathbb{K} = \mathbb{C} \end{cases} \Rightarrow \Im \mathfrak{I} \mathfrak{K} = \Re \mathfrak{I} \mathfrak{K} = 0 \Rightarrow \mathfrak{I} \mathfrak{K} = 0$$

$$\overline{\mathfrak{I} \mathfrak{K}^* \mathfrak{I} \mathfrak{K}} \leq \overline{\mathfrak{I} \mathfrak{K}} \overline{\mathfrak{I} \mathfrak{K}^*}$$

$$\overline{\mathfrak{I} \mathfrak{K}^* \mathfrak{I} \mathfrak{K}} = \overline{\mathfrak{I} \mathfrak{K}} \overline{\mathfrak{I} \mathfrak{K}^*} \xrightarrow{\text{treu}} \mathfrak{I}: \mathfrak{I} \text{ lin abh}$$

$$\mathfrak{I} \mathfrak{K}^* = 0 \xrightarrow{\text{Lem}} \mathfrak{I} \mathfrak{K} = 0 = \mathfrak{I} \mathfrak{K}^*$$

$$\begin{aligned} \mathfrak{I} \mathfrak{K} > 0 &\Rightarrow 0 \leq \overline{(\mathfrak{I} \mathfrak{K}^* - \mathfrak{I} \mathfrak{K}) \mathfrak{K}} \overline{(\mathfrak{I} \mathfrak{K}^* - \mathfrak{I} \mathfrak{K})} \\ &= \overline{\mathfrak{I} \mathfrak{K}^* \mathfrak{I} \mathfrak{K}} \overline{\mathfrak{I} \mathfrak{K}^* - \mathfrak{I} \mathfrak{K}} - \overline{\mathfrak{I} \mathfrak{K}^* \mathfrak{I} \mathfrak{K}} - \overline{\mathfrak{I} \mathfrak{K}^* \mathfrak{I} \mathfrak{K}} + \overline{\mathfrak{I} \mathfrak{K}^* \mathfrak{I} \mathfrak{K}} \\ &= \overline{\mathfrak{I} \mathfrak{K}^* \mathfrak{I} \mathfrak{K}} \overline{(\mathfrak{I} \mathfrak{K}^* - \mathfrak{I} \mathfrak{K})} \Rightarrow \overline{\mathfrak{I} \mathfrak{K}^* \mathfrak{I} \mathfrak{K}} - \overline{\mathfrak{I} \mathfrak{K}^* \mathfrak{I} \mathfrak{K}} \geq 0 \end{aligned}$$

$$\overline{\mathfrak{I} \mathfrak{K}^* \mathfrak{I} \mathfrak{K}} = \overline{\mathfrak{I} \mathfrak{K}} \overline{\mathfrak{I} \mathfrak{K}^*} = 0 \Rightarrow \overline{(\mathfrak{I} \mathfrak{K}^* - \mathfrak{I} \mathfrak{K}) \mathfrak{K}} \overline{(\mathfrak{I} \mathfrak{K}^* - \mathfrak{I} \mathfrak{K})} = 0 \xrightarrow{\text{treu}} \mathfrak{I} \mathfrak{K}^* - \mathfrak{I} \mathfrak{K} = 0$$

$$\overline{\mathfrak{I} \mathfrak{K}^*} \leq \overline{\mathfrak{I} \mathfrak{K}}$$

$$\|\cdot\| = \sqrt{\langle \cdot, \cdot \rangle} \text{ halbnorm}$$

$\langle \cdot, \cdot \rangle$ treu \Rightarrow norm

$$\|\alpha\|^2 + \|\beta\|^2 - \|\alpha + \beta\|^2 = 2\langle \alpha, \beta \rangle - \|\alpha + \beta\|^2 \geq 2\langle \alpha, \beta \rangle - \|\alpha\|^2 - \|\beta\|^2 \geq 0$$

$$\|\alpha\|^2 = \langle \alpha, \alpha \rangle = \sum_{\alpha \in \mathbb{R}} \alpha_i^2 = \|\alpha\|_{\mathbb{R}}^2$$

$$\|\alpha + \beta\|^2 + \|\alpha - \beta\|^2 = 2\|\alpha\|^2 + 2\|\beta\|^2$$

$$\text{LHS} = \|\alpha\|^2 + \|\beta\|^2 + \|\alpha + \beta\|^2 + \|\alpha - \beta\|^2 = 4\|\alpha\|^2 + 4\|\beta\|^2 = \text{RHS}$$

$$4\|\alpha\|^2 = \begin{cases} \|\alpha + \beta\|^2 + \|\alpha - \beta\|^2 & \mathbb{K} = \mathbb{R} \\ \sum_k i^k \|\alpha + \beta\|^2 + \|\alpha - \beta\|^2 & \mathbb{K} = \mathbb{C} \end{cases}$$

$$\begin{aligned} \|\alpha + \beta\|^2 + \|\alpha - \beta\|^2 &= 2\|\alpha\|^2 + 2\|\beta\|^2 \stackrel{\mathbb{K} \equiv \mathbb{R}}{\text{symm}} 4\|\alpha\|^2 \\ \sum_k i^k \|\alpha + \beta\|^2 + \|\alpha - \beta\|^2 &= \sum_k i^k \|\alpha + \beta\|^2 + i^{-k} \|\alpha - \beta\|^2 + \|\alpha - \beta\|^2 \\ &= \sum_k i^k \|\alpha + \beta\|^2 + \sum_k i^{2k} \|\alpha - \beta\|^2 + \sum_k \|\alpha - \beta\|^2 = 4\|\alpha\|^2 \end{aligned}$$