Differentialgeometrie und Analysis
Fachbereich Mathematik und Informatik
Philipps-Universität Marburg
27.03. - 30.3.2017

1. MARBURGER ARBEITSGEMEINSCHAFT MATHEMATIK (MAM)

(Non)-existence of complex structures on S^6

(1) Almost complex structures on spheres

Using characteristic classes and Bott periodicity one can prove that the only spheres that may admit almost complex structures are S^2 and S^6.

References: [Kir47, Hop48, Fri78, MT91, Mur09, Hat09, Pos91]

Speaker: Maurizio Parton

Contact person in Marburg: Panagiotis Konstantis

Talks: 1-2

(2) S^6 as a nearly Kähler manifold

Nearly Kähler structures are one of the 16 classes of almost Hermitian structures discovered by Gray and Hervella. The sphere S^6 carries a nearly Kähler structure, whose almost complex structure can be described using octonions or spinors.

References: [Cal58, BFGK91, Fri06, Agr06, Mur09, FH15]

Speaker: Aleksandra Borówka

Contact persons in Marburg: Ilka Agricola, Stefan Vasilev

Talks: 1

(3) Orthogonal complex structures near the round S^6

An almost complex structure on a Riemannian manifold is *orthogonal* if it is an isometry on each tangent space. In [LeB87], LeBrun showed that an orthogonal almost complex structure on the round S^6 is never integrable. This results was genereralized in [BHLS07] as follows: if g is a Riemannian metric on S^6 belonging to a certain neighbourhood of the round metric, then (S^6, g) does not admit an orthogonal complex structure.

References: [LeB87, Mus89, Sal96, BHL99, BHLS07, Wil16]

Speaker: Ana Cristina Ferreira, Boris Kruglikov

Contact person in Marburg: Oliver Goertsches

Talks: 2-3

(4) Dolbeault cohomology and Frölicher spectral sequence

Dolbeault cohomology and the Frölicher spectral sequence are standard topics in complex geometry. Dolbeault cohomology combines the study of differential forms on a manifold with the presence of a complex structure. The Frölicher spectral sequence measures how far is Dolbeault cohomology from de Rham cohomology.

References: [GH78, Voi02, Huy05]

Speaker: Benedikt Meinke

Contact persons in Marburg: Giovanni Bazzoni, Sönke Rollenske

Talks: 1

(5) Hodge numbers of a hypothetical complex structure on S^6

The Hodge numbers of a complex manifold are computed by Dolbeault cohomology. Under the hypothesis that S^6 has a complex structures, one can compute the corresponding Hodge numbers. In particular, although S^6 is simply connected, the Hodge number $h^{0,1}$ is non-zero for a hypothetical complex structure.

References: [Gra97, Uga00]

Speaker: Daniele Angella

Contact person in Marburg: Giovanni Bazzoni

Talks: 1-2

(6) Algebraic dimension and automorphism group of a hypothetical complex structure on S^6

The algebraic dimension of a complex manifold is the transcendence degree of the field of meromorphic functions over $\mathbb{C}$. Since the Euler characteristic of S^6 is non-zero, it follows that the algebraic dimension of a hypothetical complex S^6 should be zero. Moreover, a hypothetical complex S^6 is not almost homogeneous, i.e. the group of holomorphic automorphisms does not have an open orbit.

References: [Hop48, CDP98, HKP00]

Speaker: Christian Lehn, Caren Schinko

Contact persons in Marburg: Giovanni Bazzoni, Sönke Rollenske

Talks: 3

(7) The exceptional Lie group G_2

The exceptional Lie group G_2 and its properties will be discussed.

References: [Cal58, Sal03, Agr08]

Speaker: Cristina Draper Fontanals

Contact person in Marburg: Ilka Agricola

Talks: 1

(8) Chern's contribution

In 2003, S.-s. Chern began a study of almost complex structures on S^6, with the idea of exploiting the special properties of its well-known almost complex structure invariant under the exceptional group G_2. He proved a significant identity that solves the question for an interesting class of almost complex structures.

References: [Bry14]

Speaker: Aleksy Tralle, Markus Upmeier

Contact person in Marburg: Thomas Friedrich

Talks: 2

(9) Further approaches to the (non)-existence

In the last part we discuss different approaches to a positive, or negative, solution to the Hopf problem.

References: [Ati16, Ete15a, Ete15b, Ete15c]

Speaker: Ben Anthes, Tim Kirschner

Contact persons in Marburg: Ilka Agricola, Thomas Friedrich

References

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