Sophus Lie Seminar at Nordfjordeid, Norway
June 12-18, 2022

Titles and Abstracts of talks

Arends Christian, Paderborn, Germany

Spectral Correspondences for Rank One Locally Symmetric Spaces - The Case of Exceptional Parameters.

Abstract. There is a close connection between the Laplace spectrum for rank one compact locally symmetric spaces and the first band Ruelle-Pollicott resonances of the geodesic flow on its sphere bundle. This program was started by Flaminio and Forni for hyperbolic surfaces, continued by Dyatlov, Faure and Guillarmou for real hyperbolic spaces and by Guillarmou, Hilgert and Weich for general rank one spaces. Except for the case of hyperbolic surfaces a countable set of exceptional spectral parameters always left untreated since the corresponding Poisson transforms are neither injective nor surjective. We use vector valued Poisson transforms to treat also the exceptional spectral parameters. For surfaces the exceptional spectral parameters lead to discrete series representations of $\mathrm{SL}(2,\mathbb{R})$. In general, the resulting representations turn out to be the relative discrete series representations for associated non-Riemannian symmetric spaces.

Bertram Wolfgang, Université de Lorraine, France

On the geometry of Jordan and Lie structures.

Abstract. The title of the talk takes up the one of my "old" publication in Springer LNM 1754, see the link (http://wolfgang.bertram.perso.math.cnrs.fr/WBHabilitation.pdf). I have been working on the topic of "integrating" Jordan algebras, and related algebraic structures, since then, for more than 20 years, and I have published a more definite version (https://arxiv.org/abs/1308.5888) in 2013 (appeared in Journal of Lie Theory 24 (2014) 1067-1113). So far I have never given a talk on this definite version, probably because I feared that it might appear too abstract for a "working mathematician" : indeed, following advice of Ottmar Loos, I tried to keep this approach as general as possible, in two directions, towards infinite dimensional geometries, by working over base rings instead of fields, and towards the smallest building blocks, by allowing also the case of characteristic

2. During the last years, I have continued to work on these geometries and I have found examples making appear this approach indeed very natural, closely related to classical geometry. I shall present some of the examples and of the underlying ideas.

Claerebout Sam, Ghent University, Belgium

A minimal representation for the exceptional Lie superalgebra $D(2,1;\alpha)$.

Abstract. We construct two infinite-dimensional irreducible representations for $D(2,1;\alpha)$: a Schrödinger model and a Fock model. Further, we also introduce an intertwining isomorphism. These representations are similar to the minimal representations constructed for the orthosymplectic Lie supergroup and for Hermitian Lie groups of tube type. The intertwining isomorphism is the analogue of the Segal-Bargmann transform for the orthosymplectic Lie supergroup and for Hermitian Lie groups of tube type.

Cupit-Foutou Stéphanie, Ruhr-University of Bochum, Germany

On momentum polytopes of wonderful manifolds.

Abstract. As for toric symplectic manifolds, wonderful symplectic manifolds can be classified by some convex polytopes. After having introduced these objects, I will outline the classification.

Dahmen Rafael, Karslrusche Institute of Technology, Hermany

On the Topology of J-Groups.

Abstract. We introduce the concept of a topological J-group and determine for many important examples of topological groups if they are topological J-groups or not. Besides other results, we show that the underlying topological space of a pathwise connected topological J-group is weakly contractible which is a strong and unexpected obstruction that depends only on the homotopy type of the space.

Ferreira Ana Cristina, University of Minho, Portugal

Geodesic completeness of pseudo-Riemannian and holomorphic metrics on Lie groups.

Abstract. In this talk we will discuss geodesic completeness of left-invariant metrics for real and complex Lie groups. We will start by establishing the Euler-Arnold formalism in the holomorphic setting. We will present a new method for reobtaining the well-known classification for the real Lie group $SL(2,R)$ and, as a new addition, how it can be used to investigate the maximum domain of definition of every single geodesic for every possible metric. We will also discuss the notion of geodesic completeness for holomorphic metrics and establish a full classification for the Lie group $SL(2,C)$ for which it can be seen that holomorphic complete metrics are rare.

Glöckner Helge, Universität Paderborn, Germany

Loop groups and Birkhoff decompositions.

Abstract. Recall that a unital, associative locally convex topological algebra A over the field of complex numbers is called a continuous inverse algebra (or cia) if its group of invertible elements is open in A and the inversion map is continuous. We say that a cia A is an R-cia if it is a subalgebra of the algebra of continuous complex-valued functions on the complex unit circle S and the set of complex rational functions with all poles outside S is dense in A. This concept generalizes the R-algebras used by I. C. Gohberg and I. A. Fel'dman in the context of Wiener-Hopf equations (the latter are commutative Banach algebras with the corresponding properties).

Extending the coefficients of a finite-dimensional complex Lie algebra by an R-cia which is a complete locally convex space, we get a topological Lie algebra to which a suitable loop group can be associated. Examples include the Wiener algebra of complex-valued continuous functions on S with absolutely summable Fourier coefficients (which is an R-algebra), the Fréchet algebra of all smooth complex-valued functions on S, and the Silva algebra of all real-analytic complex-valued functions on S. We discuss Birkhoff decompositions for the corresponding loop groups, and also for the group $O(C^*, G)$ of all holomorphic functions from the punctured plane to a linear complex Lie group G, endowed with the infinite-dimensional Lie group structure constructed by Neeb and Wagemann. Some results remain valid if G is a complex Banach-Lie group.

Goertsches Oliver, Philipps-Universität Marburg, Germany

Recent results on the GKM correspondence.

Abstract. Generalizing the Delzant correspondence, which associates to a toric symplectic manifold its momentum polytope, the GKM correspondence associates to a more general class of torus actions on smooth manifolds a labelled graph. We ask the question in how far Delzant's theorem admits generalizations to this setting. This is joint work with Panagiotis Konstantis and Leopold Zoller.

Gregorovic Jan, TU Wien, Austria

Constructions of examples of almost hypercomplex/quaternionic skew-Hermitian structures.

Abstract. This talks provides several constructions of examples of almost hypercomplex/quaternionic skew-Hermitian structures. We will start with classification of quaternionic skew-Hermitian structures on semisimple symmetric spaces and derive from them a functorial construction from a category of almost quaternionic structures to a category of almost quaternionic skew-Hermitian structures. Then we also describe some more direct constructions almost hypercomplex skew-Hermitian structures.

This talk is based on joint works with I. Chrysikos and H. Winther.

Iohara Kenji, Claude Bernard University in Lyon 1, France

Elliptic root systems: classification and its properties.

Abstract. An elliptic root system (ERS) R, introduced by K. Saito in 1985, is a "root system" which is a subset of a finite dimensional real vector space F equipped with a symmetric bilinear form I of signature $(l, 2, 0)$ $(l > 0)$. A pair (R, G) of an ERS R and a reasonable one-dimensional subspace G of the radical $\mathrm{rad}(I)$ is called a marked elliptic root system (MERS). In this talk, the classification of the MERS will be explained in the case i) R/G is reduced, by K. Saito (1985), and ii) R/G is non-reduced, by A. Fialowski, K. I. and Y. Saito, with some key ideas. Some properties of their Weyl groups and automorphism groups will be given.

Loiudice Eugenia, University of Greifswald, Germany

GKM actions on cohomogeneity one manifolds.

Abstract. We consider compact manifolds M with a cohomogeneity one action of a compact Lie group G such that the orbit space M/G is a closed interval. For T a maximal torus of G, we find necessary and sufficient conditions on the group diagram of M such that the T-action on M is of GKM type, and describe its GKM graph. We apply our method to numerous examples; thus we easily establish for instance which of the cohomogeneity one manifolds with a fixed point, or of dimension up to six are of GKM type.

This is joint work with Oliver Goertsches and Giovanni Russo.

Mancini Manuel and La Rosa Gianmarco, Palermo, Italy

Two-step nilpotent Leibniz algebras.

Abstract. Leibniz algebras were first introduced by J.-L. Loday in [3] as a non-antisymmetric version of Lie algebras, and many results of Lie algebras have been extended to Leibniz algebras. Earlier, such algebraic structures had been considered by A. Blokh, who called them D-algebras. Nowdays Leibniz algebras play a significant role in different areas of mathematics and physics.

In this talk we give the classification of two-step nilpotent Leibniz algebras over a field $\mathbb{F}$, with $char(\mathbb{F}) \neq 2$, in terms of Kronecker modules associated with pairs of bilinear forms. We show that there are only three classes of nilpotent Leibniz algebras with one-dimensional commutator ideal, which we call the *Heisenberg Leibniz algebras* $\mathfrak{l}^A_{2n+1}$, parametrized by the dimension $2n+1$ and a $n \times n$ matrix A in canonical form, the *Kronecker Leibniz algebras* $\mathfrak{k}_n$ and the *Dieudonné Leibniz algebras* $\mathfrak{d}_n$, both parametrized by their dimension only.

Then we describe the Lie algebras of derivations of this class of Leibniz algebras and we show that every *almost inner derivation* (cf. [4] and [5]) of a nilpotent Leibniz algebra with one-dimensional commutator ideal, with three exceptions, is an inner derivation.

Moreover, using the Leibniz algebras / Lie local racks correspondence (see [6] and [7]), we show that nilpotent real Leibniz algebras have always a *global* integration. As an example, we integrate the indecomposable nilpotent real Leibniz algebras with one-dimensional commutator ideal.

Finally we show that every Lie quandle integrating a Leibniz algebra is induced by the conjugation of a Lie group and the Leibniz algebra is the Lie algebra of that Lie group.

References

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Paradan Paul-Emile, University of Montpellier, France

Moment polytopes in real symplectic geometry.

Abstract. In this talk, we present new results in the study of convex cones associated to isotropic representations of symmetric spaces. We explain how the inequalities describing them are parametrized by cohomological conditions: we improve here previous results due to M. Kapovich, B. Leeb and J. Millson. We detail the case of the singular Horn cone which is the counterpart of the classical Horn cone, where the eigenvalues of Hermitian square matrices are replaced by the singular values of the rectangular matrices.

Schneider Eivind, University of Tromsø, Norway

Finding ODEs that are invariant under a given Lie algebra of vector fields.

Abstract. Through examples I will discuss the problem of finding ODEs and ODE systems that are invariant under a given Lie algebra of vector fields. We approach this problem by considering the prolonged Lie algebra action on appropriate jet spaces. While the generic ODEs can be given in terms of scalar absolute differential invariants, the task of finding all invariant ODEs requires us to also compute relative differential invariants, conditional differential invariants and vector-valued differential invariants. This talk is based on joint work with B. Kruglikov and the examples are taken from the recent preprint *ODEs whose symmetry groups are not fiber-preserving.*

Stecker Leander, University of Hamburg, Germany

3-(α, δ)-Sasaki manifolds and strongly positive curvature.

Abstract. We consider 3-(α, δ)-Sasaski manifolds, generalizing the classic 3-Sasaki case. We show how these are closely related to various types of quaternionic Kähler orbifolds via connections with skew-torsion. Making use of this relation we show that in dimension 7 many such manifolds have strongly positive curvature.

Joint work with I.Agricola (Marburg) and G. Dileo (Bari).

Vega Molino Gianmarco, University of Bergen, Norway

Gauss-Bonnet theorems induced by sub-Riemannian structures.

Abstract. The classical Gauss-Bonnet theorem is a cornerstone of Riemannian geometry in which it is shown that the Euler characteristic of a manifold can be computed from integration of Riemannian curvature. This says, remarkably, that topology can be detected from a choice or Riemannian metric. This idea has since been studied and generalized in myriad ways. In the context of sub-Riemannian geometry, one works with a degenerate Riemannian structure; it is then interesting to ask if it is still possible to similarly obtain topological information.

H-type foliations are a broad class of step 2 sub-Riemannian manifolds that have an H-type group structure on the tangent space at each point; a collaboration with F. Baudoin and E. Grong has shown that it is possible to recover a Gauss-Bonnet theorem by consideration of the heat kernel of the sub-Laplacian.

An important class of H-type foliations are contact manifolds, in which one has a copy of the Heisenberg group at each tangent space. In collaboration with E. Grong and J. Hidalgo-Calderón we have considered surfaces embedded into contact manifolds equipped with the natural sub-Riemannian structure in which we recovered a Gauss-Bonnet theorem using a classical approach.

Winther Henrik, Masaryk University, Czech Republic

Symmetries of complex submanifolds of quaternionic manifolds.

Abstract. The generalized Feix construction (made by A.Borowka and D.Calderbank) identifies c-projective manifolds with type $1,1$ curvature with the totally complex submanifolds of quaternionic manifolds.

Our goal is to investigate the relationship between c-projective symmetries and quaternionic symmetries in this setting. We will show when a symmetry can be lifted to the quaternionic manifold. Moreover, we show that any maximally- or sub-maximally symmetric quaternionic manifold arises from the construction, for some c-projective manifold. In particular, a (sub-)maximally symmetric c-projective manifold can be used to construct a (sub-)maximally symmetric quaternionic manifold.

This is a joint work with A. Borowka.

Wolf Lasse, University of Paderborn, Germany

Absence of principal eigenvalues for higher rank locally symmetric spaces.

Abstract. Given a hyperbolic surface of infinite volume it is due to Patterson that there are no embedded eigenvalues of the Laplacian in the essential spectrum $[1/4, \infty)$. In my talk I will explain a generalization of this result on locally symmetric spaces of higher rank. The main example of geometries where our theorem applies are quotients of higher rank symmetric spaces w.r.t. images of Anosov representations.