Sophus Lie Seminar at Nordfjordeid, Norway
June 12-18, 2022

Titles and Abstracts of plenary lectures

Alexandre Afgoustidis, Nancy, France

Contractions of representations of real reductive groups, and the Mackey—Higson bijection.

Abstract. Let G be a real reductive group, and K be a maximal compact subgroup.

The Mackey—Higson bijection is a natural one-to-one correspondance between the irreducible admissible representations of G and those of its Cartan motion group $G_0 = K \ltimes (\mathfrak{g}/\mathfrak{k})$.

The bijection maps the tempered dual of G onto the unitary dual of G_0. Its existence was first suggested by George Mackey in the early 1970s, in part motivated by quantum-mechanical considerations based on the existence of a continuous family $(G_t)_{t\in[0,1]}$ interpolating between $G = G_1$ and G_0.

The merits of Mackey's idea appeared only in the early 1990s, when Alain Connes and Nigel Higson pointed outs its connection with the Baum–Connes conjecture for G. Progress on the Mackey bijection came in the late 2000s for complex groups thanks to Higson's efforts, and more recently for real groups.

My talk will describe the bijection, what is known about its properties and its applications, and try to outline some of the open questions in the subject.

Alexey Bolsinov, Loughborough, Great Britain

Nijenhuis Geometry.

Abstract. This talk is an introduction to Nijenhuis Geometry, a new challenging area in Differential Geometry that studies local and global properties of geometric structures given by a field ofendomorphisms with vanishing Nijenhuis torsion. This topic is located on the crossroad of Geometry, Mathematical Physics and Algebra as Nijenhuis structures naturally appear in many seemingly unrelated research areas such as bi-Hamiltonian integrable systems (both finite and infinite-dimensional), projective geometry, theory of left-symmetric algebras and others. In terms of ideas, methods and tasks, Nijenhuis geometry

most of all resembles Poisson geometry, a field of mathematics whose systematic development was initiated by Sophus Lie in the second volume of his famous "Theorie der Transformationsgruppen".

Ioannis Chrysikos, University of Hamburg, Germany

An introduction to almost hypercomplex/quaternionic skew-Hermitian structures.

Abstract. This talks provides a short introduction to the differential geometry of 4n-dimensional manifolds admitting a $\mathrm{SO}^*(2n)$-structure, or a $\mathrm{SO}^*(2n)Sp(1)$-structure, where $\mathrm{SO}^*(2n)$ denotes the quaternionic real form of $\mathrm{SO}(2n, C)$. We will explain why such G-structures form the symplectic analog of the better understood almost hypercomplex/quaternionic Hermitian structures, and hence it is reasonable to call them almost hypercomplex/quaternionic skew-Hermitian structures, respectively. In particular, we will describe the basic data encoding such geometric structures, and then focus on their intrinsic torsion and related 1st-order integrability conditions. Some examples will be also discussed, if time permitted.

This talk is based on joint works with J. Gregorovič and H. Winther

Erlend Grong, Bergen, Norway

Isometries, model spaces and curvature of sub-Riemannian manifolds.

Abstract. Sub-Riemannian manifolds are manifolds were we only have an inner product on a subbundle of the tangent bundle. There have been several recent advances regarding local geometric structures on such manifolds using representation theory and Cartan geometry.

We will discuss how such tools can give an understanding of what the sub-Riemannian analogues to spheres and hyperbolic spaces are, as well as describing sub-Riemannian flatness theorems.

Ines Kath, Greifswald, Germany

Compact quotients of the oscillator group and their spectra.

Abstract. The subject of the talk is a contribution to harmonic analysis of compact solvmanifolds. We consider the four-dimensional oscillator group Osc_1, which is a semi-direct product of the three-dimensional Heisenberg group and the real line. We classify the lattices of Osc_1 up to inner automorphisms of Osc_1 and we study the decomposition of the right regular representation $L^2(L\backslash\mathrm{Osc}_1)$ of Osc_1 into irreducible unitary representations for a lattice $L \subset \mathrm{Osc}_1$. This decomposition allows the explicit computation of the spectrum of the wave operator on the compact locally-symmetric Lorentzian manifold $L\backslash\mathrm{Osc}_1$. As a further application we mention consequences for the spectrum of the cubic Dirac operator on $L\backslash\mathrm{Osc}_1$.

The most part of the project is joint work with Mathias Fischer, the part on the cubic Dirac operator is joint work with Margarita Kraus.

Karl-Hermann Neeb, Erlangen, Germany

Geometric aspects of the modular theory of operator algebras.

Abstract. Modular theory is an important aspect of the theory of operator algebras and in the theory of local observables in Algebraic Quantum Field Theory (AQFT). It creates a one-parameter group of modular automorphisms from a single state and, sometimes, this group represents the flow of time (the dynamics) in a space-time domain. We study this question from a Lie group perspective by asking questions like: Which one-parameter groups of Lie groups can arise in this context as modular groups? This leads us to real standard subspaces of a complex Hilbert space, to antiunitary representations and to nets of standard subspaces on causal symmetric spaces.

This is joint work with Gestur Olafsson (Baton Rouge) and Vincenzo Morinelli (Rome)

Yurii Neretin, Vienna, Austria

Action of overalgebras in Plancherel decompositions.

Abstract. In non-commutative harmonic analysis there are many explicit integral transformation, which can be considered as higher analogs of Fourier transform. By our conjecture, they usually admit operational calculus. The images of certain natural differential operators of first order are differential-difference operators, usually they include differentiations of high order and difference operators include shift in imaginary direction as $f(x) \mapsto f(x+i)$, where $i^2 = -1$, and $f \in L^2(\mathbb{R})$.

Now it is known a zoo of solved problems of this type, among which, the simplest are the Fourier transform on the group $GL(2, \mathbb{R})$ and the Fourier transform on Lobachevski plane. The most advanced known statement corresponds to restrictions from $GL(n, \mathbb{C})$ to $GL(n-1, \mathbb{C})$.

Eldar Straume, NTNU, Trondheim

Sophus Lie and Felix Klein in the early 1870s; their collaboration seen in the light of their letter correspondence.

Abstract. In this historical talk I first recall some events in the 1990s, in the wake of the celebration of the 150th anniversary of the birth of Sophus Lie. This also led to the establishment of the Sophus Lie Conference Center here in Nordfjordeid, and moreover, a comprehensive Sophus Lie Archive was set up at the National Library in Oslo. Of particular value and interest was a collection of letters, still unpublished, generously received in 1997 from Lie's granddaughter Ragna Hoelder in Germany. The collection consists of Felix Klein's letters to Sophus Lie during 1870–1877, the early years of their career. Lie and Klein came to Berlin in the fall 1869 and met for the first time in late October, at the Berlin Mathematics Club. Here begins the story of an extraordinary relationship, both personally and scientifically. At this time they shared much of the same interests, namely geometrical problems in the realm of projective geometry, and Plucker's line geometry in particular. During the following 3 years they worked on joint projects, besides their more personal things, and at this time they had an intense letter correspondence. The year 1872 was particularly important for both of them; Lie became professor in Christiania (Oslo) and Klein in Erlangen. Klein presented his famous Programm in October 1872, and Lie was actually visiting him in Erlangen at that time. However, in 1873 their interests began to diverge from each other. Klein rather saw himself as a successor, or perhaps a mediator, of Riemann and Clebsch, leading him into Riemann surfaces, function theory, and the Clebschian approach to algebraic geometry. Lie worked steadily on his invariant theory of contact transformations and first order partial differential equations, which in 1873/74 gave him insight leading to his theory of continuous groups. We shall give examples to illustrate their new directions of interest. But we shall hardly touch upon what happened after the 1870s.

Michele Vergne, Paris, France

Equivariant index of Dirac operators.

Abstract. Let G be a compact Lie group acting on a Hamiltonian way on a compact symplectic manifold with Kostant line bundle L. We give a formula for the semi-classical asymptotics of the equivariant index of the Dirac operator on M tensored with powers of L. When M is the projective space, the formula coincides with the Euler-MacLaurin formula. We indicate some application of this formula to geometric quantization of possible non compact Hamiltonian manifolds.

This is a joint work with Yiannis Loizides and Paul-Emile Paradan.

Oksana Yakimova

On the Feigin–Frenkel centre and its applications to quantisation problems.

Abstract. Let G be a complex reductive group, set $\mathfrak{g} = \mathrm{Lie}\, G$. The algebra $\mathcal{S}(\mathfrak{g})^{\mathfrak{g}}$ of symmetric $\mathfrak{g}$-invariants and the centre $\mathcal{Z}(\mathfrak{g})$ of the enveloping algebra $\mathcal{U}(\mathfrak{g})$ are polynomial rings in $\mathrm{rk}\, \mathfrak{g}$ generators. There are several isomorphisms between them, including the symmetrisation map ϖ, which exists also for the Lie algebras $mathfrakq$ with $\dim \mathfrak{q} = \infty$.

However, in the infinite dimensional case, one may need to complete $\mathcal{U}(\mathfrak{q})$ in order to replace $\mathcal{Z}(\mathfrak{q})$ with an interesting related object. Roughly speaking, the Feigin–Frenkel (FF-) centre arises as a result of such completion in case of an affine Kac–Moody algebra. First we will discuss the type-free role of the symmetrisation map in the description of the FF-centre and present explicit formulas for its generators in types B, C, D, and G_2.

A 'quantisation problem' asks whether a given Poisson-commutative subalgebra $A \subset \mathcal{S}(\mathfrak{q})$ has a quantisation (a lift to a commutative subalgebra of $\mathcal{U}(\mathfrak{q})$). Several quotients of the FF-centre help to solve quantisation problems, in particular, in case of a Mishchenko—Fomenko subalgebra.

Hans Zanna Munthe-Kaas

Lie Butcher series in geometry and applications.

Abstract. B-series are Taylor series indexed by trees, originating in the seminal work of John Butcher half a century ago, and with roots going 150 years back to Arthur Cayley. It has been the main tool for analysing structure preservation in numerical integration of differential equations, and has found applications in other areas such as rough path theory and renormalisation. The modern understanding is that B-series is a canonical expansion for mappings respecting Euclidean symmetries, based on pre-Lie algebras as the algebra of canonical connections on Euclidean spaces.

In recent year B-series have been generalised to special algebras of affine connections on a manifold. Goals of this research has so far been to understand all the special algebras arising in the cases of invariant connections, where the torsion and the curvature are parallel. This yields Lie-Butcher (LB)-series for flows on Lie groups and general Klein geometries.

The general goal of defining similar expansions expansions for more general connections, such as e.g. the Levi—Civita connection of Riemannian geometries, seemed for a long time to be outside reach, but is now finding a nice solution in terms of postLie algebras. This opens a lot of possibilities for computational approaches to Riemannian and other affine geometries, such as numerical integration algorithms, rough paths and Chen-type characterisations of flow compositions.

In the talk we will give a survey of classical and recent results, as well as interesting work in progress.

Bent Ørsted, Aarhus University, Denmark

Conformal differential geometry.

Abstract. Conformal differential geometry has seen great developments since the early days of transformation groups and vector fields; the modern view involves parabolic geometry and the related calculus of connections and tractors. It also has applications to physical theories such as the AdS/CFT correspondence in field theory. In this lecture we shall survey the conformal geometry of hypersurfaces in a Riemannian manifold and the remarkable invariants found by Gover and Waldron, generalizing the Q-curvatures of Branson. We shall also present some different approaches to these quantities as found by Chang et al., and in addition report on recent joint work with A. Juhl. The celebrated Willmore invariant for a surface in Euclidian three–space is one of the simplest examples. One further aspect of the theory is the link to representations of the conformal group of the standard sphere.