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DFG

DFG-Projekt: “Multivariate Wavelet Analysis II”, SPP 1114

General Guidelines:

- Do the superiorities of general scalings really count in practice?
- Can the weak points of wavelet algorithms be ameliorated by using frames?

Construction of Multiwavelets for General Scalings

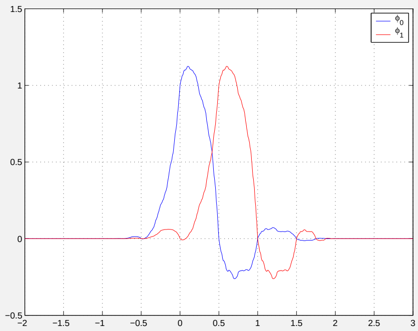
Multiwavelet bases are constructed by means of an m-scaling vector $\Phi = (\phi_0, \dots, \phi_{m-1})^T$ satisfying a matrix refinement equation

$$\Phi(x) = \sum_{k \in \mathbf{Z}^d} A_k \Phi(Mx - k), \qquad A_k \in \mathbf{R}^{m \times m},$$

where $M \in \mathbf{Z}^{d \times d}$ is an expanding integer scaling matrix. As for the scalar case, for various applications interpolating scaling vectors, i.e.

$$\phi_n(M^{-1}k) = \delta_{\rho_n,k}, \qquad \mathbf{Z}^d / M\mathbf{Z}^d \hat{=} \{\rho_0, \dots, \rho_{m-1}\},$$

with additional (bi-)orthogonality properties are advantageous. In the first period of SPP 1114, univariate orthonormal interpolating scaling vectors have been constructed. Our aim is to extend this approach to general scalings and to design Φ as smooth and localized as possible.



Orthonormal and interpolating 2-scaling vector with Sobolev regularity 0.9777 and support $[-1, 2]$

Construction of Banach Frames

In many applications, unconditional (wavelet) bases are not optimal due to their lack of flexibility. One alternative would be the use of frames, as, e.g., provided by the FEICHTINGER/GRÖCHENIG theory. Quite recently, a first generalization of this approach to manifolds has been derived by DAHLKE/STEIDL/TESCHKE. This construction starts with a continuous representation U of a locally compact group $\mathcal{G}$. After restricting U to a suitable quotient group $\mathcal{G}/\mathcal{P}$, the associated wavelet transform V_ψ and the corresponding coorbit spaces M_p

$$V_\psi f(h) := \langle f, U(\sigma(h)^{-1})\psi \rangle, \qquad M_p := \{f : \ V_\psi f \in L_p(\mathcal{G}/\mathcal{P})\}, \qquad \sigma : \mathcal{G}/\mathcal{P} \to \mathcal{G},$$

can be defined. Then, for a suitable set $(h_i)_{i \in I} \in \mathcal{G}/\mathcal{P}$, one obtains an atomic decomposition

$$f = \sum_{i \in I} c_i U(\sigma(h_i)^{-1})\psi, \quad \text{where} \quad A_1 \|f\|_{M_p} \le \|(c_i)_{i \in I}\|_{\ell_p} \le A_2 \|f\|_{M_p}.$$

In this project, we shall generalize this approach to smoothness spaces obtained as coorbit spaces associated with weighted L_p spaces. We intend to derive concrete numerical schemes for nontrivial manifolds and to establish new relationships between the classical smoothness spaces such as Besov and modulation spaces.

Applications in Image Processing

The aim is to investigate the potential of frames for the development of efficient denoising strategies. We shall focus on the generalization of the approach derived by DEVORE et al. which is based on approximation theory and relies on the solutions of certain variational problems. We plan to extract suitable variational problems also for the frame case and to find the associated optimal shrinkage parameter, at least for the white noise case. We expect the role of Besov spaces in the wavelet case to be taken over by certain coorbit spaces.

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http://www.math.uni-bremen.de/zetem/DFG-Schwerpunkt/