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DFG-Projekt: "Multivariate Wavelet Analysis III", SPP 1114

General Guidelines:

- To which extent do the superiorities of general scalings really count in practice?
- Can the weak points of wavelet algorithms be ameliorated by using frames?

Construction of Multiwavelets for General Scalings

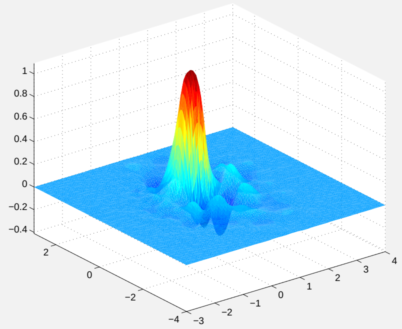
Multiwavelet bases are constructed by means of an m-scaling vector $\Phi = (\phi_0, \ldots, \phi_{m-1})^T$ satisfying a matrix refinement equation

$$\Phi(x) = \sum_{k \in \mathbf{Z}^d} A_k \Phi(Mx - k), \qquad A_k \in \mathbf{R}^{m \times m},$$

where $M \in \mathbf{Z}^{d \times d}$ is an expanding integer scaling matrix. As in the scalar case, for various applications interpolating scaling vectors, i.e.

$$\phi_n(M^{-1}k) = \delta_{\rho_n,k}, \qquad \mathbf{Z}^d / M\mathbf{Z}^d \mathrel{\hat=} \{\rho_0, \ldots, \rho_{m-1}\},$$

are advantageous. In the second period of SPP 1114, multivariate orthonormal interpolating scaling vectors have been constructed. Our aim is to extend this approach to the biorthogonal case and to incorporate additional properties such as a high order of approximation, symmetry, and high regularity.



First component of an orthonormal interpolating 2-scaling vector with Sobolev regularity 2.19

Construction of Banach Frames

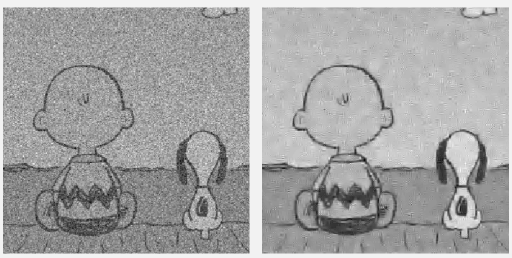
In many applications, unconditional bases are not flexible enough. One alternative is the use of frames, as, e.g., provided by the FEICHTINGER/GRÖCHENIG theory. In the second period of SPP 1114, a first generalization of this approach to manifolds has been derived. This construction starts with a continuous representation U of a locally compact group $\mathcal{G}$. After restricting U to a suitable quotient group $\mathcal{G}/\mathcal{P}$ by means of a section $\sigma : \mathcal{G}/\mathcal{P} \to \mathcal{G}$, the associated wavelet transform $V_\psi := \langle f, U(\sigma(h)^{-1})\psi \rangle$ and the corresponding weighted coorbit spaces $M_{p,w} := \{f : V_\psi f \in L_{p,w}(\mathcal{G}/\mathcal{P})\}$ can be defined. Then, for a suitable set $(h_i)_{i \in I} \in \mathcal{G}/\mathcal{P}$, one obtains an atomic decomposition

$$f = \sum_{i \in I} c_i U(\sigma(h_i)^{-1})\psi, \quad \text{where} \quad A_1 \|f\|_{M_{p,w}} \leq \|(c_i)_{i \in I}\|_{\ell_{p,w}} \leq A_2 \|f\|_{M_{p,w}} .$$

Our aim is to derive concrete numerical schemes for nontrivial manifolds and to continue our work on new relationships between the classical smoothness spaces such as Besov and modulation spaces. Moreover, we want to investigate to which extent localization properties of frames can be exploited.

Applications in Image Processing

We intend to investigate the potential of multiwavelets for general scalings and of frames for the development of efficient denoising strategies. In the second period of SPP 1114 it has turned out that multiwavelet/frame algorithms usually outperform classical wavelet schemes. In the last period, we shall focus on the generalization of the approach derived by DEVORE et al. which is based on approximation theory and relies on the solutions of certain variational problems. Our aim is to extract suitable variational problems also for the multiwavelet/frame case and to find the associated optimal shrinkage parameter. We shall study quite general kinds of noise such as colored noise and we want to include local adaptation strategies.



Denoising by means of frame shrinkage algorithms

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http://www.math.uni-bremen.de/zetem/DFG-Schwerpunkt/