Convergence of FE Based Schemes for the Euler Equations via Dissipative Weak Solutions

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January 16, 2023

3.2.2023

Since the Cauchy problem for the complete Euler system is in general ill-posed in the class of admissible weak entropy solutions, the question of convergence analysis of numerical schemes is of fundamental importance. In this context, the concept of dissipative weak (DW) solutions seems to be quite a promising approach to analyze this system analytically and numerically. In [2], the authors have studied the convergence of a class of entropy dissipative finite volume (FV) schemes for the barotropic and complete compressible Euler equations in the multidimensional case, and proved suitable stability and consistency properties to ensure convergence of their FV schemes to DW solutions. The theory has been further developed for several (classical) FV schemes [3] where recently also convergence towards DW solutions have been proven for high-order finite element (FE) based schemes [1, 4].
In this talk, we give an introduction to the concept of DW solutions and summarize recent results inside this framework. We consider, in particular, high-order FE schemes and prove convergence to a DW solutions in the multidimensional case. To this end, it is crucial that structure preserving properties, such as positivity preservation and entropy inequality hold, and the schemes are consistent with the underlying PDE. We show how to ensure them and in numerical simulations, we verify our theoretical findings, cf. Figure 1. Finally, since the concept of DW solutions is not restricted to the Euler case, we give an outlook of further developments.

Acknowledgements

M.L.-M. and P.O. gratefully acknowledge support of the Gutenberg Research College, JGU Mainz.

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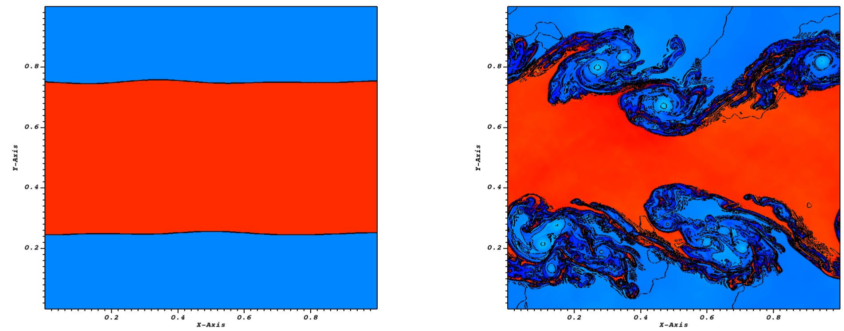

Figure 1: Kelvin-Helmholz instability.

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