

# Mini workshop on group actions in symplectic and Kähler geometry

	WEDNESDAY	THURSDAY
09:00–10:00	Dmitri Panov	Silvia Sabatini
10:00–10:30	coffee break	coffee break
10:30–11:30	Nicholas Lindsay	Isabelle Charton
11:30–13:30	lunch	lunch
13:30–14:30	Nikolas Wardenski	Bart Van Steirteghem
14:30–15:00	coffee break	coffee break
15:00–16:00	Leopold Zoller	Yael Karshon

## Titles and Abstracts

### Isabelle Charton

Title: *Tall and monotone complexity one spaces of dimension six*

Abstract: Let  $(M^{2n}, \omega)$  be a compact symplectic manifold of dimension  $2n$ . Assume that a  $(n - 1)$ -dimensional torus  $T^{n-1}$  acts effectively on  $(M^{2n}, \omega)$  in a Hamiltonian fashion with moment map  $\phi: M^{2n} \rightarrow \text{Lie}^*(T^{n-1})$ . Then  $(M^{2n}, T^{n-1}, \omega, \phi)$  is called a complexity-one space.

In this talk, we focus on a specific class of complexity-one spaces, namely those which are tall and monotone. We give a complete classification of these spaces for  $n = 3$  and show that the  $T^2$ -action can be extended to an effective Hamiltonian  $T^3$ -action in that case.

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### Yael Karshon

Title: *Complexity one Hamiltonian torus actions*

Abstract: I will report on my classification, joint with Sue Tolman, of Hamiltonian torus actions with two dimensional quotients.

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## Nicholas Lindsay

Title: *TBA*

Abstract: TBA

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## Dmitri Panov

Title: *Symplectic Fanos and their symmetries*

Abstract: This survey talk is based on joint papers with Joel Fine and Nick Lindsay. I'll explain what are symplectic Fanos, and will give the only two known constructions of them (via Algebraic geometry and via twistor spaces of hyperbolic manifolds). Next a conjecture about the classification of 6-dimensional Fanos with Hamiltonian  $S^1$ -actions will be discussed. Finally, it will be explained that Tolman's symplectic manifold induces an exotic (non-algebraic) symplectic structure on a  $\mathbb{C}\mathbb{P}^1$ -bundle over  $\mathbb{C}\mathbb{P}^2$ . .

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## Silvia Sabatini

Title: *Some topological properties of monotone complexity one spaces*

Abstract: In symplectic geometry it is often the case that compact symplectic manifolds with large group symmetries admit indeed a Kähler structure.

For instance, if the manifold is of dimension  $2n$  and it is acted on effectively by a compact torus of dimension  $n$  in a Hamiltonian way (namely, there exists a moment map which describes the action), then it is well-known that there exists an invariant Kähler structure. These spaces are also called complexity-zero spaces, where the complexity is given by  $n$  minus the dimension of the torus. In this talk I will explain how there is some evidence that a similar statement holds true when the complexity is one and the manifold is monotone (the latter being the symplectic analog of the Fano condition in algebraic geometry), namely, that every monotone complexity-one space is simply connected and has Todd genus one, properties which are also enjoyed by Fano varieties.

These results were largely inspired by a conjecture posed by Fine and Panov in 2010, by work of Lindsay and Panov of 2019, and it is joint with Daniele Sepe.

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## Bart Van Steirteghem

Title: *The root system of a multiplicity free Hamiltonian manifold*

Abstract: I will discuss some of the information encoded in the root system  $F$ . Knop has associated to a compact connected multiplicity free Hamiltonian manifold  $M$  and will describe an algorithm for computing it using the momentum polytope of  $M$ . This talk is based on joint work with G. Pezzini.

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**Nikolas Wardenksi**

Title: *Equivariant Gompf gluing and its applications*

Abstract: We generalize the gluing construction of Robert Gompf by carrying it to the equivariant category. Moreover, we show that this is reverse to symplectic cutting and use it to determine the isomorphism type of certain complexity one manifolds in dimension 6.

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**Leopold Zoller**

Title: *Realization of GKM fibrations and new examples of Hamiltonian non-Kähler actions*

Abstract: In the mid 90s, Tolman gave a remarkable example of a Hamiltonian torus action in dimension 6 which does not admit a compatible Kähler structure. We will analyse this example from the point of view of GKM theory, which associates to certain manifolds a labelled graph. In dimension 6, it turns out that the graph determines the manifold up to diffeomorphism, which lets us identify Tolman's example as Eschenburg's twisted flag. Conversely, we give a construction which realizes graph fibrations in dimension 6 through projectivizations of complex vector bundles. As a consequence, we see that Tolman's example is part of a large family of manifolds whose graphs enjoy similar properties. By relating properties of the graphs to geometric structures, we find many more examples of Hamiltonian non-Kähler actions. This is joint work with Oliver Goertsches and Panagiotis Konstantis

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