Wavelet Detection of Periodic Behavior in EEG and ECoG Data

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ABSTRACT

This paper deals with the analysis of electrical potential time series derived from brain activity of patients before and during an epileptic seizure. Outstanding problems for this analysis include the prediction and space-time localization of seizures. We shall focus on the prediction problem, whose satisfactory solution would provide maximal lead time in which to predict an epileptic seizure.[1–4] In particular, we shall present a fast method aimed at detecting periodic behavior inherent in EEG seizure data.

The proposed procedure recognizes interindividual different periodic behavior in the electrical brain activity during an epileptic seizure. The method is composed of three steps:

1. ECoG data of an individual patient are analyzed through spectral and wavelet methods to extract periodic patterns associated with epileptic seizures of a specific patient.

2. Using this knowledge of seizure periodicity, we construct an *optimal piecewise constant wavelet* designed to detect the epileptic periodic patterns of the patient.

3. We introduce a fast discretized version of the continuous wavelet transform, as well as waveletgram averaging techniques, to detect occurrence and period of the seizure periodicities in the preseizure EEG data of the patient. The algorithm is formulated to provide real time implementation.

Our procedure is generally applicable to detect locally periodic components in signals s which can be modelled as

$$s(t) = A(t)F(h(t)) + N(t) \quad \text{for} \quad t \quad \text{in} \quad I, \tag{1}$$

where F is a periodic signal, A is a nonnegative slowly varying function, and h is strictly increasing with h' slowly varying. N denotes background activity. In the case of ECoG data, N is essentially $1/f$ noise. In the case of EEG data and for t in I, N includes noise due to cranial geometry and densities.[5,6] In both cases N also includes standard low frequency rhythms.[7]

PIECEWISE CONSTANT WAVELETS

We shall introduce the notion of piecewise constant wavelets, and give a constructive method for choosing the optimal piecewise constant wavelet with which to detect periodicities of a known shape. Our wavelet transformation is an $L^p(\mathbf{R})$ normalized discrete variation of the continuous wavelet transformation

$$W_F(b,a) = a^{-1/p} \int_{\mathbf{R}} F(t)\psi(\frac{t-b}{a})dt = a^{-1/p} \int_{\mathbf{R}} F(t)\psi_{b,a}(t)dt.$$

This discretized version of W_F was introduced by Benedetto and Colella[1], but we shall see that a solution of our period detection and computation problem requires wavelets that were not implemented in that work. The inherent redundancy in our wavelet transform reduces the effects of some noises; and the expected computational complexity of the process is significantly reduced through the restriction to piecewise constant wavelets.

Definition: *A piecewise constant wavelet of degree M is a function $\psi \in L^2(\mathbf{R})$ such that*

$$\int_{\mathbf{R}} \psi(t)dt = 0,$$

$$\exists s_i \in \mathbf{R}, i = 1, ..., N \textit{ such that } \psi|_{[s_i, s_{i+1})} = c_i, \ c_i \in \mathbf{R} \textit{ for } i = 1, ..., N-1,$$

$$\psi = 0 \quad \textit{on} \ \ \mathbf{R} \setminus [s_1, s_N),$$

and

$$\exists M \in \mathbf{N} \textit{ such that } Ms_i \in \mathbf{Z} \textit{ for all } i = 1, ..., N.$$

Note that ψ has compact support, i.e., $supp(\psi) \subseteq [s_1, s_N]$ and $supp(\psi_{b,a}) \subseteq [as_1 + b, as_N + b]$. The first observation in our approach to period detection and computation is the following fact.

Theorem 1: *Let $F \in L^2(\mathbf{T}_T)$, i.e., F is T-periodic with finite energy, and let ψ be a piecewise constant wavelet of degree M. Then*

$$a^{1/p} W_F(b, a) = \int_{\mathbf{R}} F(t)\psi(\frac{t-b}{a})dt$$

is T periodic in b and MT-periodic in a.

This result implies that if the signal s has the particular form $s(t) = AF(ct)$ for constants A and c, then the relative maxima of W_s form a lattice in time-scale space. The horizontal (time) distance between two neighboring vertices of the lattice is $1/c$, and the vertical (scale) distance between two neighboring vertices is M/c.

Detecting lattice patterns and measuring the distance between vertices in redundant waveletgrams will disclose periodicities in the signal. Averaging techniques will reduce the effect of noise in the case $s(t) = AF(ct) + N(t)$.

CONSTRUCTION OF OPTIMAL PIECEWISE CONSTANT WAVELETS

We shall detect lattice patterns in time-scale space in terms of relative maxima of wavelet transforms. To do this, we begin by fixing $N \in \mathbf{N}$ and we consider piecewise constant wavelets ψ^c of the form

$$\psi^c|_{[i,i+1)} = c_i \text{ for } i = 0, ..., N-1, c = (c_0, c_1, ..., c_{N-1}) \in \mathbf{R}^N. \tag{2}$$

In particular, we have

$$0 = \int_{\mathbf{R}} \psi^c(t)dt = \sum_{i=0}^{N-1} c_i, \tag{3}$$

and we normalize ψ^c so that

$$\|\psi^c\|_{L^2(\mathbf{R})} = \|c\|_{l^2(\mathbf{R^N})} = 1. \tag{4}$$

Equation (3) allows us to achieve the periodicity properties asserted in Theorem 1. Note that (3) is equivalent to the condition that

$$c \in H = \{x \in \mathbf{R}^N : \sum_{i=0}^{N-1} x_i = \langle x, (1,1,...,1,1)\rangle = 0\}.$$

H is an $N-1$ dimensional subspace, i.e., a hyperplane. Equation (4) is a standard normalization constraint which can be made in constructing wavelets ψ^c, and it can be expressed as

$$c \in S^{N-1} = \{x \in \mathbf{R}^N : \|x\|_{l^2(\mathbf{R^N})} = 1\}.$$

Note that $M = 1$ in this setting.

The following theorem answers the general question of how to choose optimal piecewise constant wavelets:

Theorem 2: *Let $F \in L^2(\mathbf{T}_T)$ and let $N \in \mathbf{N}$.*

a. There exist $(b_0, a_0) \in \mathbf{R} \times \mathbf{R}^+$ such that

$$a_0^{-\frac{1}{p}} \|P_H(k_{b_0,a_0})\|_{l^2(\mathbf{R^N})} = \max_{(b,a)\in\mathbf{R}\times\mathbf{R}^+} a^{-\frac{1}{p}} \|P_H(k_{b,a})\|_{l^2(\mathbf{R^N})},$$

where $k_{b,a} = (k_{b,a,0}, ...k_{b,a,N-1}) \in R^N$ is defined by $k_{b,a,i} = \int_{ia+b}^{(i+1)a+b} F(t)dt$ and P_H is the orthogonal projection of $\mathbf{R}^N$ onto the hyperplane H,

$$H = \{x \in \mathbf{R}^N : \sum_{i=0}^{N-1} x_i = \langle x, (1,1,...,1,1)\rangle = 0\}$$

b. For this (b_0, a_0) we set

$$c_0 = \frac{P_H(k_{b_0,a_0})}{\|P_H(k_{b_0,a_0})\|_{l^2(\mathbf{R^N})}}.$$

The piecewise constant wavelet ψ^{c_0} satisfies (2),(3), and (4), and

$$|W_F^{\psi^{c_0}}(b_0, a_0)| \geq |W_F^{\psi^c}(b, a)|$$

for all $(b, a) \in \mathbf{R} \times \mathbf{R}^+$ and all ψ^c satisfying (2),(3), and (4).

Note that the optimization process depends on the choice of p.

Figure 1 illustrates the application of this theorem. After drawing the expected period, in our case the seizure period of an individual patient, we define the periodic function F associated with the seizure period. F is sampled at 130 samples per period for subsequent calculations with the projection P_H. We choose $N = 5$ and calculate $k(b, a) = \|P_H(k_{b,a})\|_{l^2(\mathbf{R^N})}$. For the normalization constants $p = 1$, $p = 1.35$, and $p = 2$ we obtain distinct optimal piecewise constant wavelets.

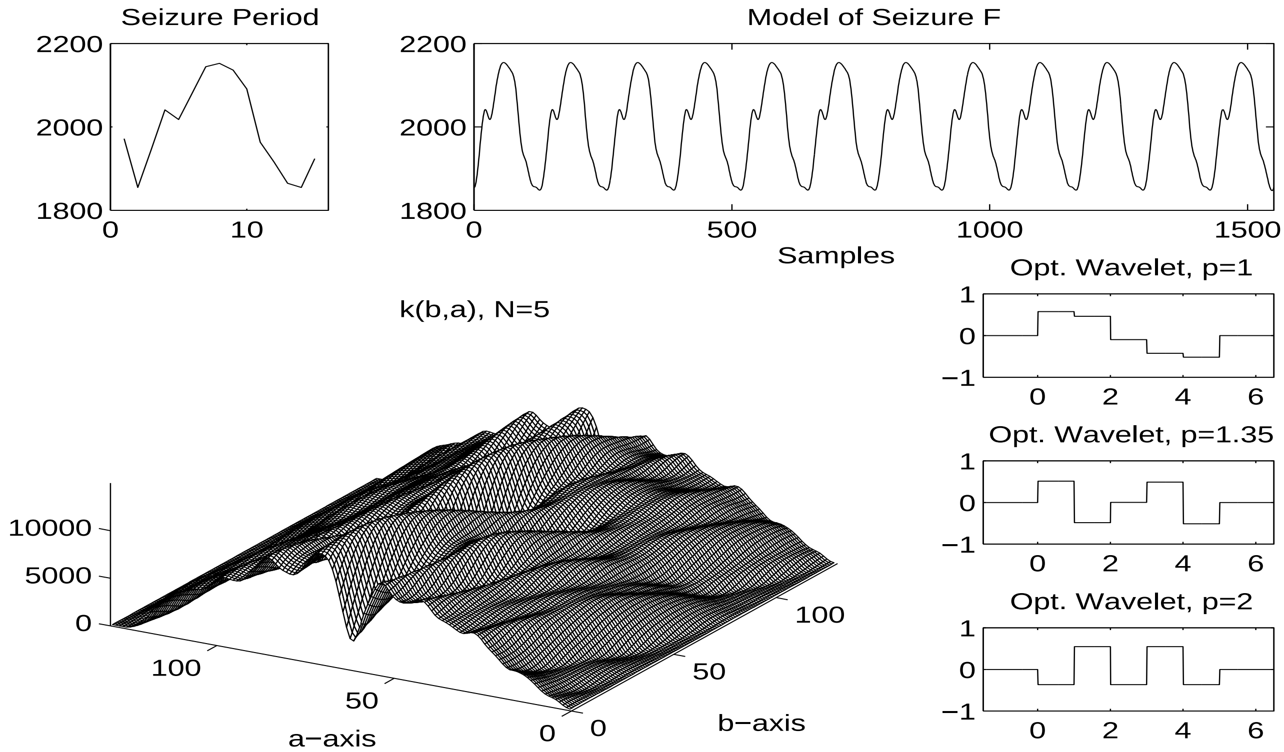

Figure 1: *Designing optimal piecewise constant wavelets to extract seizure periods.*

PERIODICITY DETECTION

The periodicities discribed in Theorem 1 allow us to introduce averaging methods in the following way in order to analyze the waveletgram. Theorem 1 implies that if F is T-periodic then $a^{\frac{1}{p}} W_F(a,b)$ takes the same value on each of the cells

$$[b_0 + iT, b_0 + (i+1)T] \times [jMT, (j+1)MT] \text{ for } i \in \mathbf{Z} \text{ and } j \in \mathbf{N_0}.$$

Consequently, for any signal s and positive integers Q and R, we define the average

$$V_s^{R,Q}(b,a,T) = \frac{1}{v^Q(a,T)(2R+1)} \sum_{r=-R}^{R} \sum_{q=0}^{Q} W_s^{\psi^c}(b + rT, a + qT),$$

where

$$v^Q(a,T) = a^{\frac{1}{p}} \sum_{q=0}^{Q} (a+qT)^{-\frac{1}{p}}, a, T \in \mathbf{R}^+,$$

with $T \in \mathbf{R}^+, a \in (0,T)$, and $b \in [0,T)$. Thus, if F is a T_0-periodic signal then

$$V_F^{R,Q}(b,a,T_0) = W_F^{\psi^c}(b,a).$$

As such, for any signal s, we set

$$Z_s^{R,Q}(T) = \sup_{a \in [0,T), b \in [0,T)} |V_s^{R,Q}(b,a,T)|. \tag{5}$$

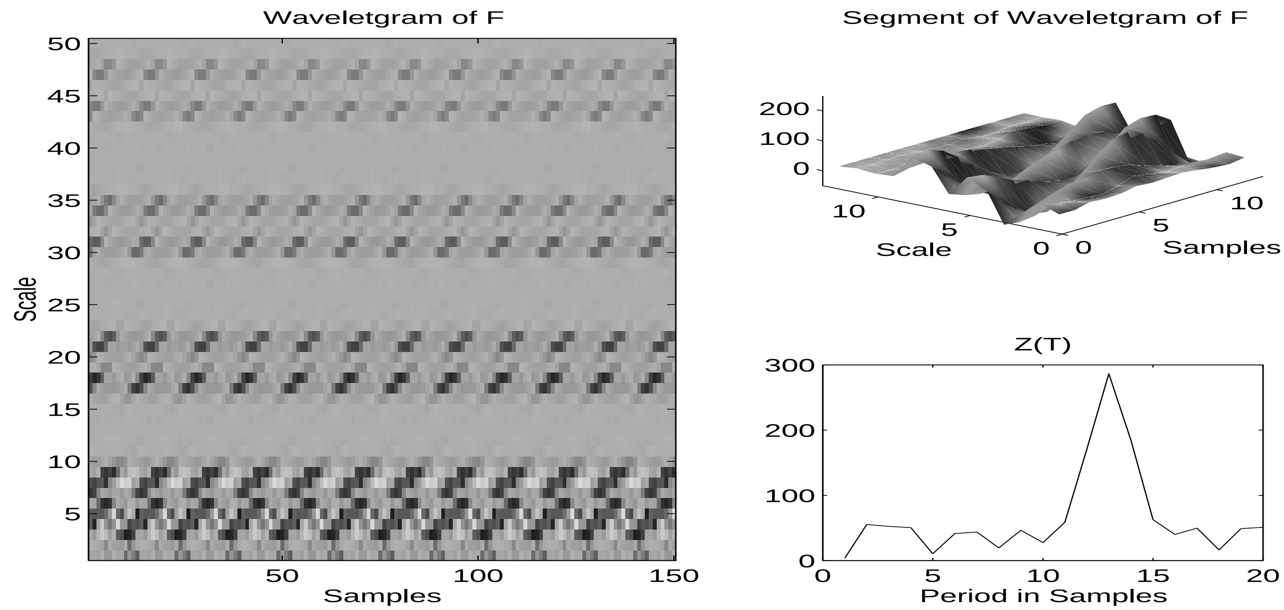

Figure 2: *Illustration of the algorithm applied for period detection. Here $p = 1.35$, N=5.*

Therefore,

$$Z_F^{R,Q}(T_0) = \sup_{a\in[0,T_0),b\in[0,T_0)} |W_F^{\psi^c}(b,a)|,$$

and we expect $Z_F^{R,Q}(T)$ to be small for $T \neq T_0$ and for large values of Q and R. Further, for several types of noises N, including white noise, this averaging criterion will give small values of $Z_N^{R,Q}(T)$ for all values of T and for large values of Q and R. Hence, the subadditivity inherent in (5) for signals $s = F + N$, combined with Theorems 1 and 2, give rise to an algorithm for computing T_0.

In Figure 2 we apply this method to the signal F constructed in Figure 1 sampled at a realistic rate of 13 samples per period. First, we obtain the $p = 1.35$ normalized wavelet transform of F. $Z_F^{R,Q}(T)$ is then calculated for $T = 1, ..., 20$. The maximum of Z in Figure 2 implies the occurence of the periodic signal with period length of 13 samples.

The technique has been successfully applied to synthesized noisy data. We are currently working with EEG and ECoG data.

Theorem 2 remains true if we replace the hyperplane H by any subspace contained in H. This fact allows us to include additional features in the optimization process of Theorem 2. For example, a desirable feature is to use wavelets with multiple vanishing moments. Thus, we ask if we can construct a subspace $U \subseteq \mathbf{R}^N$ such that ψ^c has n vanishing moments for each $c \in U$. In fact, we have proven the following result.

Theorem 3: *For $N \geq 2$ and $0 \leq n \leq N-2$, and define $v_k = (1^k, 2^k, 3^k, ..., N^k) \in \mathbf{R}^N$ for $k = 0, ..., n$. Consider the subspace*

$$U^n = span\{v_0, ..., v_n\}^{\perp}.$$

Then $c \in U^n$ if and only if ψ^c has $n + 1$ vanishing moments

REMARKS

1. The proposed method is based on knowledge of the periodic component F. Typically, this knowledge is obtained through ECoG data analysis. After gathering ECoG information about F, we expect to obtain real time detection and computation of seizure periodicities in noisy EEG data by means of the averaging method we have just described. Thus, the invasive ECoG technique will properly assume a very limited role in long term prediction studies.

2. If F is a trigonometric polynomial, then the signals described in (1) have been analyzed by Kronland-Martinet, Seip, Torresani, et al. to deal with the problem of detecting spectral lines in NMR data.[8,9] Another technique, that of computing critical frequencies in ECoG seizure data using waveletgram striations, was formulated by Benedetto and Colella[1]. These frequencies are related to the *instantaneous frequency*[10] $h'(t)$ of s at t; and, with our period detection and computation problem in mind, $1/h'(t)$ is the *instantaneous period* of s at t.

3. Our analysis of ECoG seizure data establishes that the periodic function F in (1) is not a simple trigonometric polynomial. This fact, and the goal of real time period detection and computation, has led us to the described wavelet approach.

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