Jacobs University Bremen
School of Engineering and Science
Götz Pfander, Sergei Markouski

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Analysis I — Problem Set 1

Issued: 05.09.07 Due: 11.09.07, noon, Analysis Mailbox in Research 1

1.1. Negate the following statements:

(a) $\exists\, p \in \mathbb{Q}: \quad p^2 = 2$

(b) $\forall\, p \in \mathbb{Q} \quad \exists\, n \in \mathbb{N}: \quad n \geq p$

(c) $\forall\, m \in \mathbb{N} \quad \forall\, n \in \mathbb{N}: \quad m \leq n \quad \Longrightarrow \quad m+1 \leq n+1$

1.2. Let A, B and C be sets. Determine whether the following statements are true or false. If a double implication fails, determine whether one or the other implication holds. Give a proof or counterexample respectively:

(a) $(A \cup B)^c = A^c \cap B^c$

(b) $(A \cap B)^c = A^c \cup B^c$

(c) $A \subset B \quad \wedge \quad A \subset C \quad \Longleftrightarrow \quad A \subset (B \cup C)$

(d) $A \subset B \quad \vee \quad A \subset C \quad \Longleftrightarrow \quad A \subset (B \cap C)$

1.3. (a) Denote the *identity function* for any set C by id_C, i.e. $id_C : C \longrightarrow C,\ x \mapsto x$.
Let $f : A \longrightarrow B$ be a function.
$g : B \longrightarrow A$ is called a *left inverse* for f if $g \circ f = id_A$ and
$h : B \longrightarrow A$ is called a *right inverse* for f if $f \circ h = id_B$.
Prove that

(a) f has a left inverse $\Longleftrightarrow$ f is injective.

(b) f has a right inverse $\Longleftrightarrow$ f is surjective.

(b) Show that for any finite set M the following equivalence holds:

$$f : M \longrightarrow M \text{ is injective} \quad \Longleftrightarrow \quad f : M \longrightarrow M \text{ is surjective}$$

1.4. Let A, B be ordered sets with order $\leq_A$, $\leq_B$ respectively and $(a_1, b_1), (a_2, b_2) \in A \times B$. Define

$$(a_1, b_1) \leq (a_2, b_2) :\Leftrightarrow \quad (a_1 \neq a_2 \ \wedge\ a_1 \leq_A a_2) \quad \vee \quad (a_1 = a_2 \ \wedge\ b_1 \leq_B b_2).$$

Show that $\leq$ is an order on $A \times B$.
Remark: This order on $A \times B$ is called the *dictionary order*.