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Analysis I — Problem Set 2

Issued: 10.09.07 Due: 18.09.07, 12:00 am

2.1. Let $A \neq \emptyset$ be a set. Prove the following two statements:

(a) Any partition $\mathcal{D} = \{D_i\}_{i \in I}$ of A induces on A the equivalence relation defined by $a \sim b$ if $a, b \in D_{i_0}$ for some $i_0 \in I$.

(b) Any equivalence relation $\sim$ on A can be used to define a partition $\mathcal{D}$ of A.

Hence studying partitions is the same as studying equivalence relations.

2.2. Let $(F, +, \cdot)$ be a field.

(a) Show that 0 is uniquely determined: there is only one neutral element under addition.

(b) For every $x \in F$, $-x$ is uniquely determined: there is only one $y \in F$ with $x + y = 0$.

(c) Show that for every $x, y \in F$ we have $0x = 0$; $(-x)y = -(xy)$; $(-x)(-y) = xy$.

2.3. Show that the following statements hold in an ordered field F. Remember $x > y :\Leftrightarrow (x \geq y \wedge x \neq y)$.

(a) If $x > 0$ then $0 > -x$, and vice versa.

(b) If $x > 0$ and $z > y$, then $xz > xy$.

(c) If $0 > x$ and $z > y$ then $xy > xz$.

(d) If $x \neq 0$ then $x^2 > 0$. In particular, $1 > 0$.

(e) If $y > x > 0$ then $1/x > 1/y > 0$.

2.4. Show that $\mathbb{Q}$ with the order $O_{\mathbb{Q}}$ on $\mathbb{Q}$ given in class is an ordered field. Use the definition of $\mathbb{Q}$ as equivalence classes with respect to the equivalence relation $R_{\mathbb{Q}}$.