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Fall Term 2004

Analysis I — Problem Set 9

Issued: 5.11.07 Due: 13.11.07, noon

9.1. The closure, boundary and interior of a set. Let A be a set in the metric space X. Recall that A' is the set of cluster points of A , $\overline{A}$ is the closure of A, A° is the interior of A, and ∂A is the boundary of A.
Prove or give counterexamples to the following statements:

(a) $A \subseteq \partial A, \quad \partial A \subseteq A$

(b) $A \subseteq \overline{A}, \quad \overline{A} \subseteq A$

(c) $A \subseteq A^\circ, \quad A^\circ \subseteq A$

(d) $A \subseteq A', \quad A' \subseteq A$

(e) $\partial A \subseteq \partial\partial A, \quad \partial\partial A \subseteq \partial A$

(f) $\overline{A} \subseteq \overline{\overline{A}}, \quad \overline{\overline{A}} \subseteq \overline{A}$

(g) $A^\circ \subseteq A^{\circ\circ}, \quad A^{\circ\circ} \subseteq A^\circ$

(h) $A' \subseteq A'', \quad A'' \subseteq A'$

(i) $A' \subseteq \overline{A}, \quad \overline{A} \subseteq A'$

(j) $A \cup A' \subseteq \overline{A}, \quad \overline{A} \subseteq A \cup A'$

9.2. Closed sets. Let (X, d_X) be a metric space. Show the following statements:

(a) Finite unions and arbitrary intersections of closed subsets of X are closed.

(b) $\overline{A} \cup \overline{B} = \overline{A \cup B} \ \ \forall A, B \in X$.

(c) $\cup_{i\in I}\overline{A_i} \subset \overline{\cup_{i\in I} A_i}$, where all $A_i \in X$.
Give an example when equality fails.

9.3. Zeroes of a continuous function. Let $f : K \to \mathbb{R}$, where $K \subset \mathbb{R}$ is a compact set, be a continuous function. Prove that the set of zeroes of f is a compact set in $\mathbb{R}$.

9.4. Compact sets. Prove that [0,1] is covering compact in $\mathbb{R}$.

9.5. Continuity and compactness(Bonus Problem). Let X, Y be two metric spaces and $f : X \to Y$ a continuous bijection. Prove that if X is compact, then f is a homeomorphism.